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Last Update : 20 / 11 / 2017

Article : "Improving particle beam acceleration in plasmas",
M.C.de Sousa and I.L.Caldas, Physics of Plasmas 25, 043110 (2018),
doi : 10.1063 / 1.5017508, View online : https : // doi.org / 10.1063 / 1.5017508
Preprint available : https : // arxiv.org / abs / 1710.03879

This code was used to generate Figures 1 and 2 of the article described above, and to obtain the minimum and maximum energies of particles in the islands of the main resonance.

```
(* Initial conditions (IC):
 Column 1 - theta_j,
Column 2 - Ij,
Column 3 - Type of trajectory: 0 = Perturbing Robust Barrier in Black,
1 = Separatrix in Grey, 2 = Invariant Tori in Purple,
3 = Resonant Islands in Red, 4 = Resonant Islands in Green,
5 = Chaos in Blue, 6 = Resonant Islands in Ochre *)
PathName =
  "C:\\Wave-Particle_Interactions\\Charged_Particle_in_E_Wave_and_B_Field";
ICFileName = "IC_Figure1a.dat";
Print[
  StringTake[ICFileName, {4, 9}] <> " " <> StringTake[ICFileName, {10, 10}] <>
   "(" <> StringTake[ICFileName, {11, 11}] <> ")"];
IC = Import[PathName <> "\\" <> ICFileName, "Table"];
IC[[All, {1}]] = ToExpression[IC[[All, {1}]]];
IC[[All, {2}]] = ToExpression[IC[[All, {2}]]];
IC[[All, {3}]] = ToExpression[IC[[All, {3}]]];

pllot = Table[{}, {j, 1, Length[IC]}];
NumIterationsSep = 40000;
(* Number of iterations for the separatrix *)
NumIterationsChaos = 3000;
(* Number of iterations for chaotic trajectories *)
NumIterationsOther = 8000;
(* Number of iterations for other trajectories *)
PrintIndividualTrajectories = 0;

k = 4;                  (* wave number *)
T = 2 Pi (1 + (1 / 15));     (* wave period *)
TicksBottom = Table[If[(Mod[i, π/2] < 10^-5) || (Mod[i, π/2] == π/2),
    {i, i, {0.03, 0}}, {i, "", {0.015, 0}}], {i, 0, 2 π, π/8}];

If[StringTake[ICFileName, {11, 11}] == "a",
  {eps = 0.01;     (* 2 x wave amplitude *)
   FigTitle = "(a)      ε = 0.01";
```

```
    If[StringTake[ICFileName, {10, 11}] == "1a",
     {TicksLeft =
       Table[If[(Mod[i, 0.5] < 10^-5) || (Mod[i, 0.5] == 0.5), {i, NumberForm[i,
            {3, 2}], {0.03, 0}}, {i, "", {0.015, 0}}], {i, 0, 3.50, 0.1}];
      PRange = {{-0.001, 2 Pi}, {0.0, 3.5}}}]},

   If[StringTake[ICFileName, {11, 11}] == "b",
    {eps = 0.2;      (* 2 x wave amplitude *)
     FigTitle = "(b)       ε = 0.20";

     If[StringTake[ICFileName, {10, 11}] == "1b",
      {TicksLeft =
        Table[If[(Mod[i, 0.2] < 10^-5) || (Mod[i, 0.2] == 0.2), {i, NumberForm[i,
             {3, 2}], {0.03, 0}}, {i, "", {0.015, 0}}], {i, 0, 1.00, 0.05}],
       PRange = {{-0.001, 2 Pi}, {0.0, 0.85}}}]}]];


If[StringTake[ICFileName, {10, 10}] == "1",
  ARatio = 0.95,
  If[StringTake[ICFileName, {10, 10}] == "2",
   {TicksLeft = Table[If[(Mod[i, 0.05] < 10^-5) || (Mod[i, 0.05] == 0.05),
       {i, NumberForm[i, {3, 2}], {0.03, 0}},
       {i, "", {0.015, 0}}], {i, 0, 0.30, 0.01}];
    ARatio = 0.7;
    PRange = {{-0.001, 2 Pi}, {0.0, 0.2}}}]];


Do[{
   NumIterations = If[IC[[r, 3]] == 1, NumIterationsSep,
     If[IC[[r, 3]] == 5, NumIterationsChaos, NumIterationsOther]];
   m = Table[{0, 0}, {j, 1, NumIterations}];

   thetaj = IC[[r, 1]];
   Ij = IC[[r, 2]];
   IjOne = 0.1;

   Do[{
     (* Map that describes the time evolution of the system - Eq. (6) *)
     IjOne = 0.5 ((2 Ij (Sin[thetaj])^2) +
         ((Sqrt[2 Ij] Cos[thetaj]) + (0.5 eps k Sin[k Sqrt[2 Ij] Sin[thetaj]]))^2) ,
     thetaj = Mod[ArcTan[(2 Sqrt[2 Ij] Cos[thetaj]) + (eps k Sin[k Sqrt[2 Ij]
              Sin[thetaj]]), 2 Sqrt[2 Ij] Sin[thetaj]] + T/Sqrt[1 + 2 IjOne], 2 Pi],
     Ij = IjOne,
```

```
      m[[j]] = {thetaj, Ij}}, {j, 1, NumIterations}],

    TrajectoryColor = If[IC[[r, 3]] == 0, RGBColor[0, 0, 0],
      If[IC[[r, 3]] == 1, RGBColor[0.4, 0.4, 0.4], If[IC[[r, 3]] == 2,
        RGBColor[0.6, 0, 0.6], If[IC[[r, 3]] == 3, RGBColor[1, 0, 0],
         If[IC[[r, 3]] == 4, RGBColor[0, 1, 0], If[IC[[r, 3]] == 5,
           RGBColor[0, 0, 1], If[IC[[r, 3]] == 6, RGBColor[0.8, 0.7, 0]]]]]]],
    pllot[[r]] = ListPlot[m, Ticks → {{0, π/2, π, 3π/2, 2π}, Automatic},
      PlotStyle → {PointSize[0.0005], TrajectoryColor},
      PlotRange → {{0, 2 Pi}, Automatic}],
    If[PrintIndividualTrajectories == 1, Print[pllot[[r]]]]}, {r,
    1, Length[IC]}];

Show[{Table[pllot[[r]], {r, 1, Length[IC]}]},
 PlotLabel → Style[FigTitle, "Times", 11],
 Frame → True, FrameStyle → Directive[AbsoluteThickness[1], "Times", 10.5],
 FrameLabel → {Style["θ", "Times", 11], Style["I", "Times", Italic, 11]},
 Axes → False, FrameTicks → {TicksBottom, TicksLeft, None, None},
 PlotRange → PRange, AspectRatio → ARatio]


(* Minimum and maximum velocities of particles in the
 islands of the main resonance. Particles enter and leave
 the accelerator through its central position. Therefore,
velocities are calculated in the center of the accelerator x=y=0,
which corresponds to theta=0;Pi *)
If[StringTake[ICFileName, {10, 10}] == "2",
  {NumIterations = 20000;     (* Number of iterations *)
   m = Table[{0, 0, 0}, {j, 1, NumIterations}];

   thetaj = Pi;
   IjOne = 0.1;

   If[StringTake[ICFileName, {11, 11}] == "a",
    {Ij = 0.03023;
     IndexMin = 2;
     IndexMax = 1},
    If[StringTake[ICFileName, {11, 11}] == "b",
     {Ij = 0.02930;
      IndexMin = 3;
      IndexMax = 1}]];

   Do[{
     (* Map that describes the time evolution of the system - Eq. (6) *)
     IjOne = 0.5 ((2 Ij (Sin[thetaj])^2) +
         ((√(2 Ij) Cos[thetaj]) + (0.5 eps k Sin[k √(2 Ij) Sin[thetaj]]))^2) ,
```

```
  thetaj = Mod[ArcTan[(2 √(2 Ij) Cos[thetaj]) + (eps k Sin[k √(2 Ij)
          Sin[thetaj]]), 2 √(2 Ij) Sin[thetaj]] + T/√(1 + 2 IjOne), 2 Pi],
  Ij = IjOne,
  DistancePi = Abs[thetaj - Pi],
  m[[j]] = {thetaj, Ij, DistancePi}, {j, 1, NumIterations}];

Print[ListPlot[m[[All, {1, 2}]], Ticks → {{0, π/2, π, 3π/2, 2π}, Automatic},
  PlotStyle → {PointSize[0.0005], RGBColor[1, 0, 0]},
  PlotRange → {{0, 2 Pi}, {0, 0.2}}]];

(* The function Ordering[] gives the
 positions in m[[All,{3}]] (DistancePi) at which each
 successive element of Sort[m[[All,{3}]]] appears. *)
DistancePiOrder = Ordering[m[[All, {3}]]];

Print["DistancePi at minimum energy = "];
Print[DistancePiMin = m[[DistancePiOrder[[IndexMin]], 3]]];
Print["theta at minimum energy = "];
Print[thetaMin = m[[DistancePiOrder[[IndexMin]], 1]]];
Print["Imin = "];
Print[Imin = m[[DistancePiOrder[[IndexMin]], 2]]];
Print["Emin = "];
Print[Emin = Sqrt[1 + (2 Imin)]];
Print["pMin = "];
Print[pMin = Sqrt[2 Imin] Cos[thetaMin]];
Print["vMin = "];
Print[vMin = pMin/Sqrt[1 + pMin^2]];

Print["DistancePi at maximum energy = "];
Print[DistancePiMax = m[[DistancePiOrder[[IndexMax]], 3]]];
Print["theta at maximum energy = "];
Print[thetaMax = m[[DistancePiOrder[[IndexMax]], 1]]];
Print["Imax = "];
Print[Imax = m[[DistancePiOrder[[IndexMax]], 2]]];
Print["Emax = "];
Print[Emax = Sqrt[1 + (2 Imax)]];
Print["pMax = "];
Print[pMax = Sqrt[2 Imax] Cos[thetaMax]];
Print["vMax = "];
Print[vMax = pMax/Sqrt[1 + pMax^2]]}];
```

Figure 1(a)

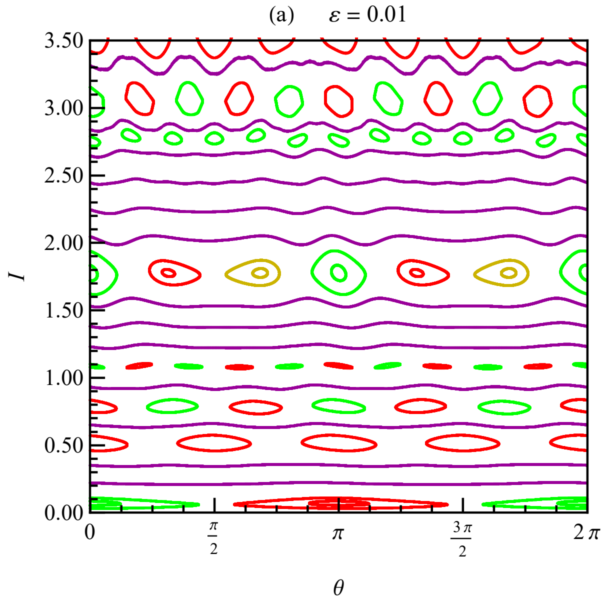


Figure 1(b)

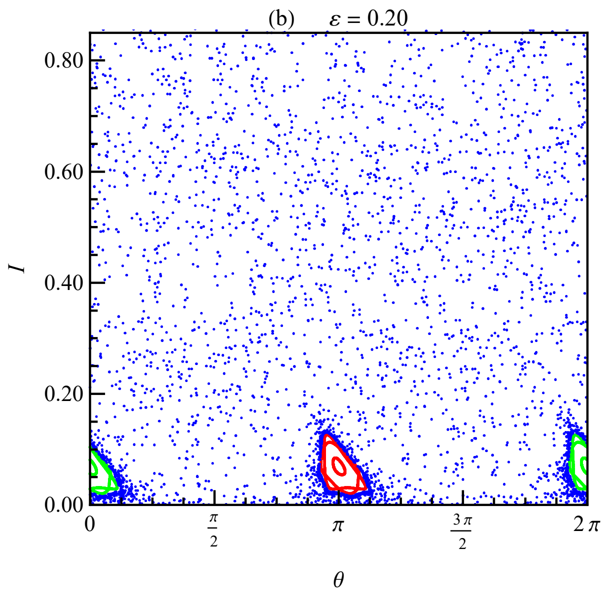
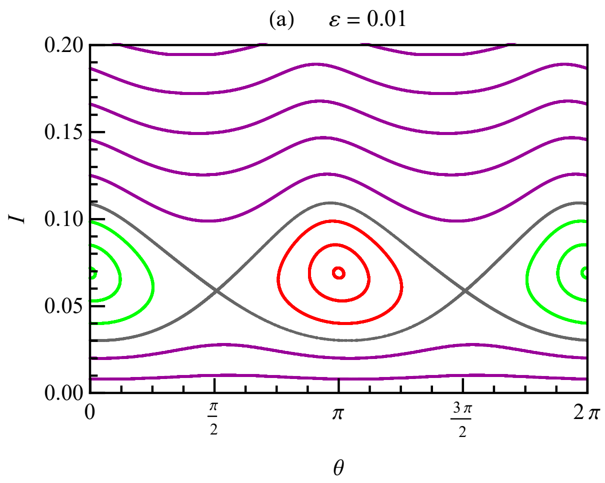
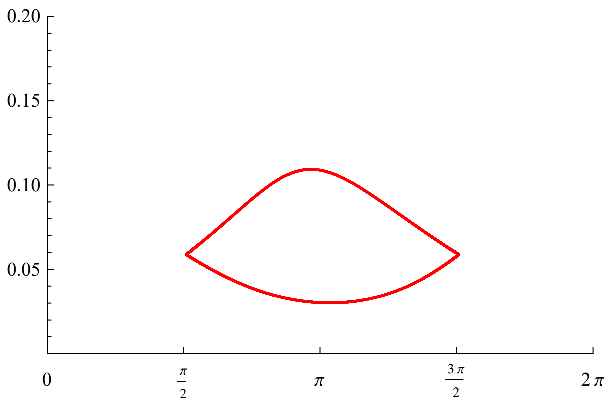


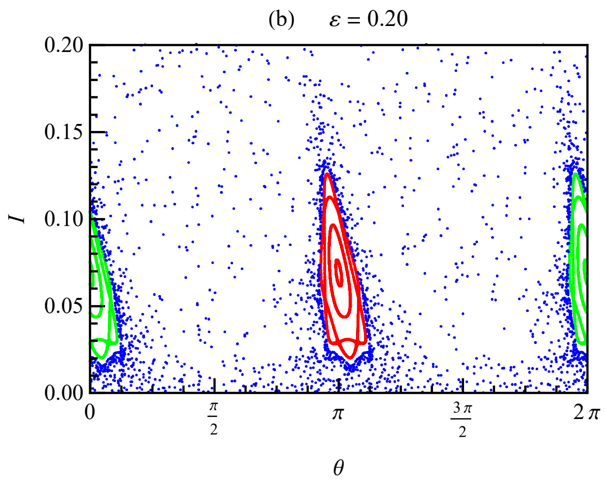
Figure 2(a)


(a) ε = 0.01
0.20
0.15
0.10
0.05
0.00
I
0
π/2
π
3π/2
2π
θ




```
DistancePi at minimum energy =
0.000292205
theta at minimum energy =
3.1413
Imin =
0.0302307
Emin =
1.02979
pMin =
-0.245889
vMin =
-0.238777
DistancePi at maximum energy =
0.000137885
theta at maximum energy =
3.14173
Imax =
0.108695
Emax =
1.10335
pMax =
-0.46625
vMax =
-0.422575
```

Figure 2(b)


(b) ε = 0.20
0.20
0.15
0.10
0.05
0.00
I
0
π/2
π
3π/2
2π
θ

DistancePi at minimum energy =

1.31289×10^{-6}

theta at minimum energy =

3.14159

Imin =

0.0293011

Emin =

1.02888

pMin =

-0.242079

vMin =

-0.235283

DistancePi at maximum energy =

3.43126×10^{-7}

theta at maximum energy =

3.14159

Imax =

0.10342

Emax =

1.09856

pMax =

-0.454798

vMax =

-0.413993