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Fractal Structures in the Basin of Attraction of Chimera States

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Introduction

Chimera states are dynamical states of coupled networks and are characterized by the existence of spatially coherent and incoherent regions simultaneously. Here we investigate these states focusing on the properties of the basin of attraction and its boundaries in a network of non-locally coupled Hénon maps with periodic boundary conditions. Our results show that the boundaries between the basins of chimera and coherent states present a fractal structure, whereas in the case of incoherent and chimera states the basins present properties of riddling. Therefore, the chimera states can be highly sensitive to changes in the initial conditions, which can impose great difficulties in predicting the final state of the network.

The Model and Results

Our model is a network composed of $N = 500$ coupled Hénon maps given by [1]

$$\mathbf{x}_{t+1}^{(i)} = \mathbf{F}(\mathbf{x}_t^{(i)}) + \frac{\sigma \mathbf{E}}{2rN} \sum_{j=i-rN}^{i+rN} [\mathbf{F}(\mathbf{x}_t^{(j)}) - \mathbf{F}(\mathbf{x}_t^{(i)})]. \tag{1}$$

Where $i = 1, \ldots, N$ is the index of each map, t is the discrete time, $\mathbf{F}(\mathbf{x}) = [1 - \alpha x^2 + y, \beta x]^T$ is the two-dimensional Hénon map, σ and r are the coupling intensity and coupling radius, respectively, and

$$\mathbf{E} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \tag{2}$$

specifies that only the x variable of each map is coupled.

To investigate the basins of attraction we fixed the initial values of all but one map of the network as 0. For the remaining map we varied its initial conditions in a grid and characterised the spatial dynamic of the resulting state by according to its value of the strength of incoherence (SI)[2].

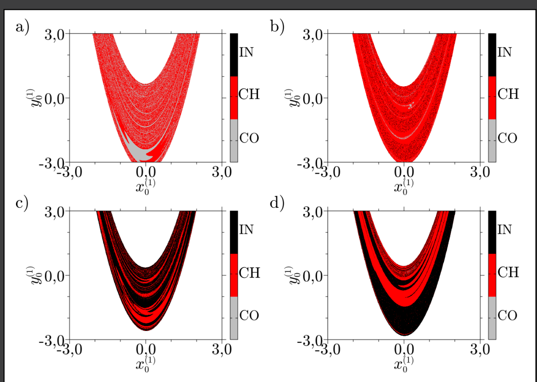

Figure 1: Basins of attraction of the network of coupled Hénon maps for coherent (CO), chimera (CH), and incoherent (IN) states. We consider σ equal to (a) 0.30, (b) 0.24, (c) 0.18, and (d) 0.12.

The Model and Results

The characterisation of basin boundaries can be made using the initial condition uncertainty fraction, as introduced by McDonald et al [3]. The fraction of uncertain points $f(\varepsilon)$ as a function of a small neighbourhood ε, is expected to scale according to $f(\varepsilon) \sim \varepsilon^{\gamma}$, where γ is the uncertainty exponent. Given the uncertainty exponent, one can easily obtain the fractal dimension using the relation $d = 2 - \gamma$ [3].

Here we start by calculating $f(\varepsilon)$ for the boundary between the chimera and coherent states basins. From the fitting of the points we obtain $\gamma = 0.30$ (red dots) $\gamma = 0.15$ (blue dots), and $\gamma = 0.02$ (green dots) for $\sigma = 0.18, 0.24$, and 0.30, respectively. As a result, the boundaries between the chimera and coherent states basins are fractal.

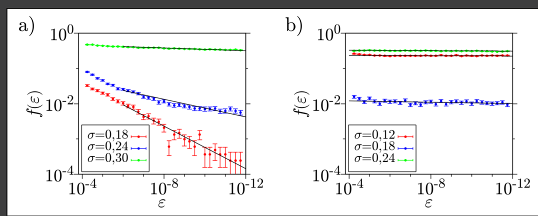

Figure 2: Uncertainty fraction $f(\varepsilon)$ versus the uncertainty radius ε for the boundary between (a) the chimera and the coherent basins, and (b) the chimera and the incoherent state basins.

Then we compute $f(\varepsilon)$ for the boundary between the chimera and incoherent states basins. In this case, our results show that $f(\varepsilon)$ remains approximately constant for different σ values and, as a consequence, $\gamma \approx 0$. Which indicates the existence of a riddled basin.

Conclusions

From the estimated value of the uncertainty exponents we conclude that the boundaries between the basin of the coherent and chimera states, present a fractal structure, whereas the boundaries between the basins of incoherent and chimera states present properties of riddling. The presence of such structures indicate that an improvement in the precision of the initial conditions may have a virtually null effect in the prediction of the final state of the network.

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