# Correlated Brownian motion and diffusion of defects in spatially extended chaotic systems

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**Resumo**

One of the spatiotemporal patterns exhibited by coupled map lattices with nearest-neighbor coupling is the appearance of chaotic defects, which are spatially localized regions of chaotic motion with a particle-like behavior. Chaotic defects display random behavior and diffuse along the lattice with a Gaussian behavior. In this note we investigate some dynamical properties of chaotic defects in a lattice of coupled chaotic quadratic maps. Using a recurrence-based diagnostic we found that the motion of chaotic defects is well-represented by a stochastic time series with a power-law spectrum, i.e. a correlated Brownian motion. The correlation exponent corresponds to a memory effect in the Brownian motion, and increases with a system parameter as the diffusion coefficient of chaotic defects.

## Mathematical Section

We use as a paradigmatic model of spatio-temporal dynamics the following coupled map lattice

$$u_{n+1}^{(i)} = (1-\epsilon) f(u_n^{(i)}) + \frac{\epsilon}{2}\left[f(u_n^{(i+1)}) + f(u_{tn}^{(i-1)})\right] \quad (1)$$

where $u_n^{(i)}$ is a state variable at discrete time $n = 0, 1, 2, ...$ and discrete position $i = 1, 2, ...N$ in a onedimensional lattice of size $N$. The system at each spatial position undergoes a temporal dynamics described by a quadratic map

$$f(u) = u + \alpha u - \beta u^2 \quad (2)$$

where $0 \leq u \leq 1$. In the following we fix $\beta = 0.1$ and use $\alpha$ as a variable parameter. We will consider random initial conditions over the interval $0 \leq u_0 \leq 1$ and fixed boundary conditions: $u_n^1 = u_n^N = 0$.

## Results

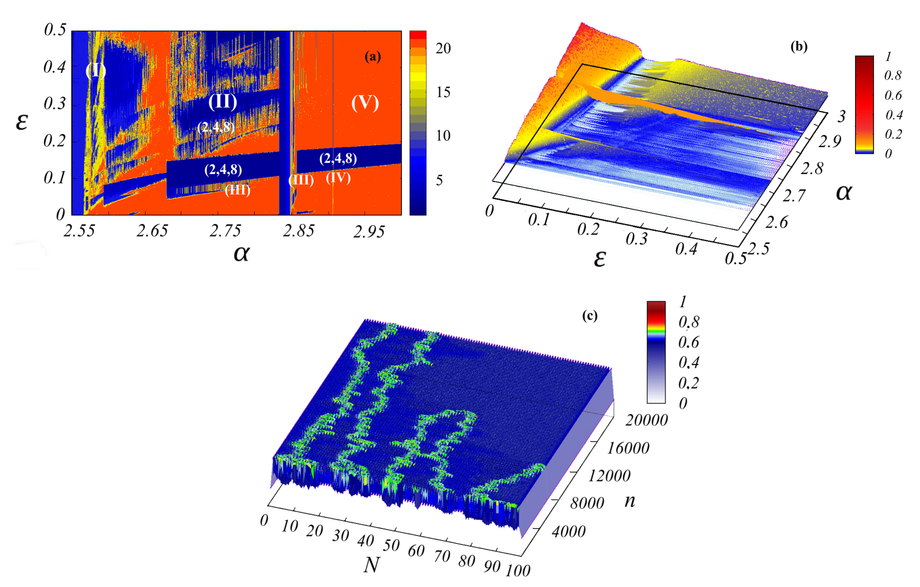

**Figura 1:** (a) Phase diagram of the coupled map lattice: it depicts the dominant temporal period of the spatio-tempora attractor for given values of $\varepsilon$ and $\alpha$, for $beta = 0.1$ and $N = 100$; (b) The density of KS entropy as a function of the same parameters; (c) Space-time plot of the coupled quadratic map lattice (1) for $\varepsilon = 0.1$ and $\alpha = 2.85$.;

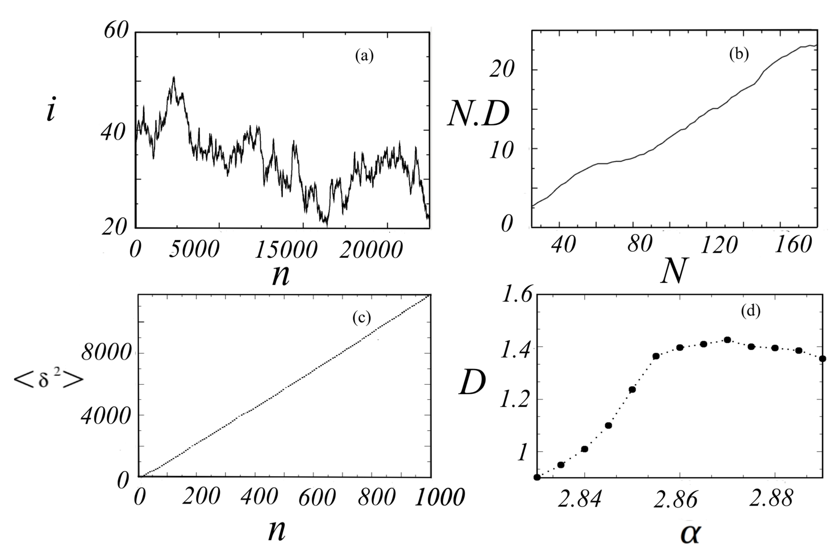

**Figura 2:** (a) Motion of an individual defect; (b) Number of chaotic defects as a function of the lattice size; (c) Mean square displacement of the chaotic defects as a function of time for a lattice of $N = 6000$ maps with $\alpha = 2.85$, $\beta = \varepsilon = 0.1$ (d) Diffusion coefficient of defects as a function of the nonlinearity parameter $\alpha$

The recurrence plot is a tool to depict recurrences of a $K$ length time series and is defined as a $K \times K$ binary matrix.

$$\mathbf{R}_{(i,j)}^{m,\epsilon_i} = \Theta(\epsilon - \|\mathbf{x}_i - \mathbf{x}_j\|), \mathbf{x}_i \in R^m, i, j = 1, ..., K \quad (3)$$

where $\epsilon$ is the vicinity parameter.
The concept of subsets of the recurrence plot has been generalized by using recurrence microstates $A(\epsilon)$, namely all possible recurrence states of two randomly selected short sequences of $N = 2, 3$, and 4 consecutive points in a $K(K \gg N)$ length time series. Such microstates $A(\epsilon)$ are $N \times N$ small binary matrices and for a large enough randomly select number of sample $M$, the recurrence entropy $S$ can be computed by.

$$S(\epsilon) = -\sum_{i=1}^{M} PA_i \ln PA_i$$

where $PA_i$ is the probability of occurrence of a specific state $A_i(\epsilon)$.

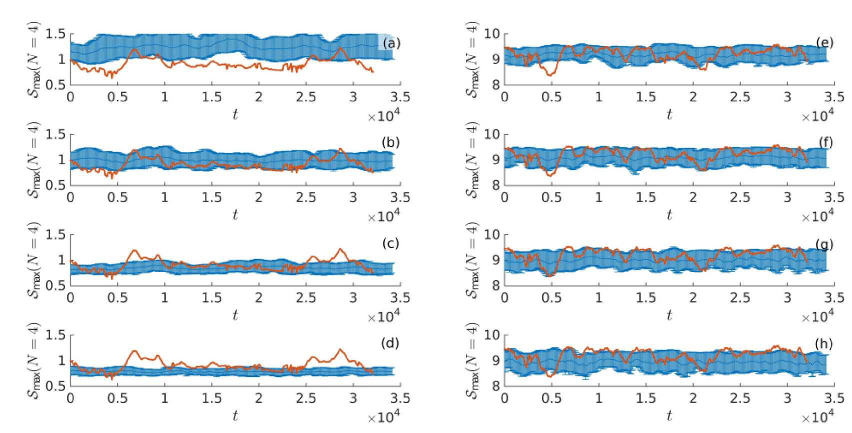

**Figura 3:** (a-d) Time series of the maximal recurrence entropy $max(S)$ computed for the defect signal (orange lines), superposed with the dynamical behavior of $max(S)$ (blue lines) computed for stochastic time correlated signals following a power law spectrum $1/f^\sigma$ , $\sigma = 2.1, 2.3, 2.5, 2.7$ respectively. Vertical lines are representative of the standard deviation of each windowed computed $max(S)$. (e-h) shuffled version of all signal.

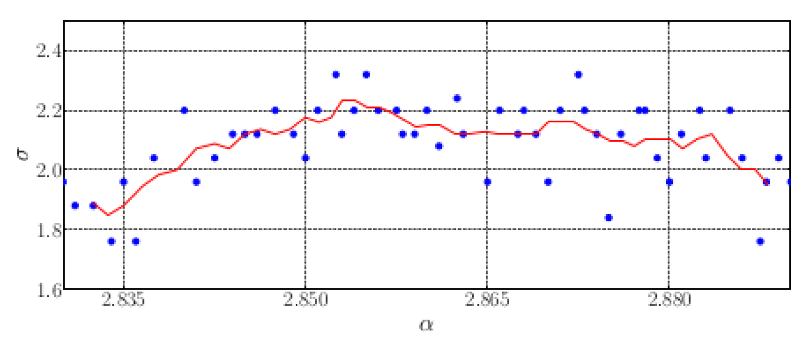

**Figura 4:** Dependence of the power-law exponent $\sigma$, of the correlated stochastic signal, with the nonlinearity parameter $\alpha$.

## Conclusions

- we have revisited the formation and dynamics of chaotic defects in lattices of coupled quadratic maps. Chaotic defects are spatially localized regions of chaotic dynamics which propagate along the lattice in a particle-like fashion. Since we have used random initial conditions in numerical simulations we believe that the formation of chaotic defects is a quite general phenomenon.
- We found that the number of defects increases with the lattice size, and that defects experience a normal (gaussian) diffusion with a diffusion coefficient around 1.25. This value was found to depend on the nonlineary parameter Îś of the coupled maps.
- The motion of individual defects is similar to a random walk, and we used a recently developed recurrence-based technique to show that it is actually a stochastic time series displaying a power-law power spectrum $1/f^\sigma$ , where $\sigma$ lies between 2.3 and 2.4. The latter exponent was also found to be dependent on the nonlinearity parameter $\alpha$. In other words, the defect motion has recurrence properties similar to a stochastic signal with memory.

## Referências

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