

## Introduction

The use of complex networks to model neural systems found a great background where the neurons are described by the nodes and their connections as the edges of the network [1]. By this modeling, it is possible to analyze the synchronization characteristics of the network, which is an important characteristic of the neural system.

Here, a network composed of  $N = 2000$  bursting thermally sensitive neurons is simulated by using a Hodgkin-Huxley-type neural model [2]. In this context, synchronization phenomena are studied as a function of the neurons' temperature and coupling strength, where bursting and spike synchronizations are observed. The mechanism that generates these behaviors may be understood as an interplay between the individual-uncoupled-neurons dynamics and the coupling influence.

## Neural model

The main equation of the model of Braun *et. al.* [2], which describes the membrane potential evolution, is given by:

$$C_m \frac{dV_i}{dt} = -J_{i,Na} - J_{i,K} - J_{i,sd} - J_{i,sa} - J_{i,L} + J_{i,coupling}, \quad (1)$$

in which  $C_m$  is the membrane capacitance,  $J_{i,v}$  is the ionic flux over the  $i$ th neuron membrane and related to Sodium (Na), Potassium (K) and leak (L) currents. Additionally, two slow fluxes due to Calcium, (sd) and (sa) are considered. The parameters of the model were chosen following [3]

The coupling term,  $J_{i,coupling}$ , is modeled by excitatory chemical synapses:

$$J_{i,coupling} = \frac{\varepsilon(V_{syn} - V_i)}{\chi} \sum_{j=1}^N e_{i,j} r_j, \quad (2)$$

where  $\varepsilon$  is the coupling strength (mS/cm<sup>2</sup>),  $\chi = 4.8$  is the average number of connections,  $V_{syn} = 20$  mV is the synaptic reversal potential, and  $r_j$  is the kinetic term [3].

## Results

Main results of synchronization and its relation to the individual-uncoupled-neural behavior for different neurons' temperature and coupling values.

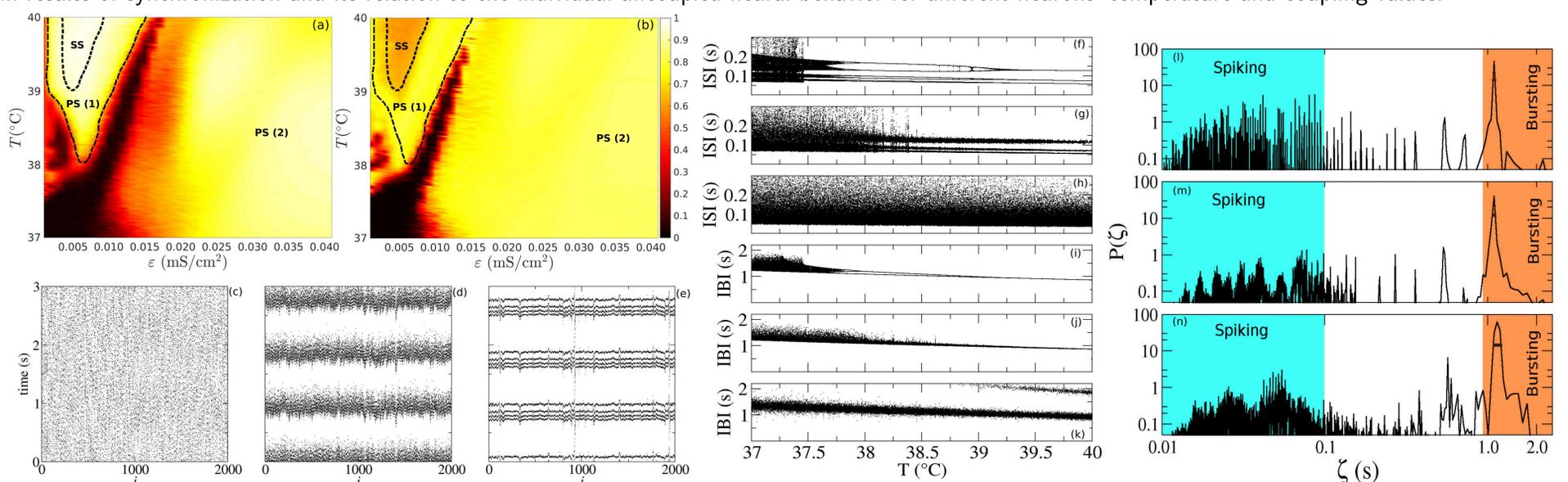


Figure 2 –  $R$  and  $\Delta$  as a function of  $T$  and  $\varepsilon$  (panels (a) and (b)) (SS - spike synchronization; PS - Phase synchronization.) Panels (c), (d), and (e) depict the raster plot of the network. Panels (f), (g), and (h) depicts the Inter-Spike intervals and panels (i), (j), and (k) depict the Inter-Burst intervals as a function of temperature. Panels (l), (m), and (n) show the Fourier transform of the individual neuron signal. The synchronization for the weak coupling regime is related to the periodic behavior of the uncoupled neuron.

## Conclusions

- Burst and spike synchronizations are observed in the parameter space of neurons' temperature and coupling;
- Increases on the neurons' temperature make the network depicts a higher synchronization level;
- The recurrence quantification analysis is able to measure spike and burst synchronization;
- The interplay between the individual behavior of neurons and the coupling results in the synchronization phenomena observed.

## Acknowledgment



## Synchronization quantifiers

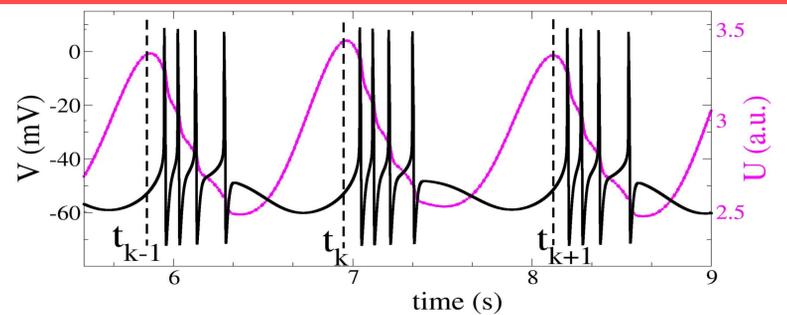


Figure 1 – Dynamics of the neural model. The burst beginning is given by the  $U$  maxima.

The Kuramoto order parameter [4] can measure phase synchronization:

$$R(t) = \left| \frac{1}{N} \sum_{k=1}^N e^{i\theta_k(t)} \right|. \quad (3)$$

Here,  $\theta$  is the phase associated with each neuron, given by:

$$\theta_i(t) = 2\pi k_i + 2\pi \frac{t - t_{k,i}}{t_{k+1,i} - t_{k,i}}, \quad t_{k,i} < t < t_{k+1,i}. \quad (4)$$

If  $R \rightarrow 1(0)$ , the network is (not) on a phase synchronized state.

Besides that, the recurrence analysis [5] is used to analyze synchronization

$$\mathbf{R}_{ij}(\mu) = \Theta(\mu - \|\mathbf{w}_i - \mathbf{w}_j\|), \quad \mathbf{w}_i \in \mathbb{R}, \quad i, j = 1, 2, \dots, S, \quad (5)$$

where  $\mu$  is the recurrence threshold and the recurrence matrix ( $\mathbf{R}$ ) is evaluated from the mean-field network time series  $\bar{V}(t)$ . In this scenario, the determinism is the ratio of the recurrent points that belong to diagonal structures:

$$\Delta(\ell_{min}, \mu) = \frac{\sum_{\ell=\ell_{min}}^S \ell P(\ell, \mu)}{\sum_{\ell=1}^S \ell P(\ell, \mu)}. \quad (6)$$

If  $\Delta \rightarrow 1(0)$ , the network is (not) on a phase synchronized state.

## References

- [1] - S. H. Strogatz, *Nature*, **410**, 268, 2001.
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