Escape basin method applied to the evaluation of the parameter space for asymmetric nontwist systems

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Abstract

Nontwist systems violate the twist condition at least one point in the phase space. The non-fulfillment of this condition gives rise to new structures in the phase space, as the twin islands chains and the presence of the shearless curve. The curves in the phase space behave as barriers for the transport of chaotic trajectories and the knowledge of when they exist is really import for the dynamical analysis. When the system fulfill a symmetry transformation, we can use the indicator points to determine when a barrier exist, more precisely, the shearless curve. Otherwise, when the system is asymmetric, we need to develop a new method to construct the parameter space for the existence of transport barrier. In this work, we study the escape basins and its applicability to determine when the transport barrier exist. This provide us a helpful way to estimate the breakup of the last shearless curve in the parameter space. We study the extended standard nontwist map, the standard nontwist map with addition of a perturbation, whose symmetry has a dependence of the parameters space values of the system.

Extended Standard Nontwist Map

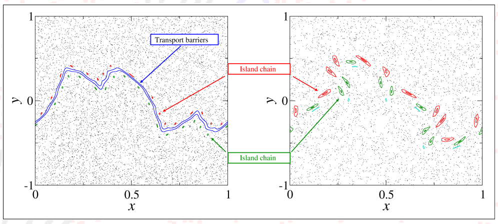
The extended standard nontwist map (ESNM) is defined by the following,

$$y_{n+1} = y_n - b\sin(2\pi x_n) - c\sin(2\pi m x_n),$$
$$x_{n+1} = x_n + a(1 - y_{n+1}^2) \mod 1,$$

where a, b and $c \in \mathbb{R}$, and m is an integer parameter.

Phase spaces

$b = 0.597$, $c = 0.005$ and $m = 2$



$a = 0.695$ $a = 0.805$

Symmetry transformation and the violation of the symmetry

A map M is symmetric over a transformation T if M satisfies the relation

$$\mathbf{TM} = \mathbf{MT}.$$

For the standard nontwist map

$$T = \left(x \pm \frac{1}{2}, -y\right).$$

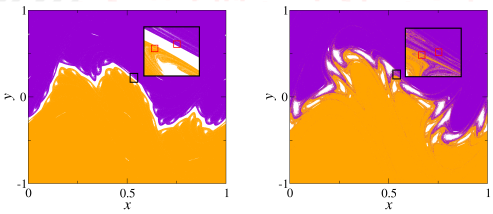
Applying the relation to the ESNM

$$(x \pm 1/2) + a(1 - y_{n+1}^2) = [x + a(1 - y_{n+1}^2)] \pm 1/2$$
$$-y + b\sin(2\pi x) + c(-1)^{m+1}\sin(2\pi m x) = -y + b\sin(2\pi x) + c\sin(2\pi m x)$$

The ESNM is symmetric only for m **odd**.

Escape basins

$b = 0.597$, $c = 0.005$ and $m = 2$



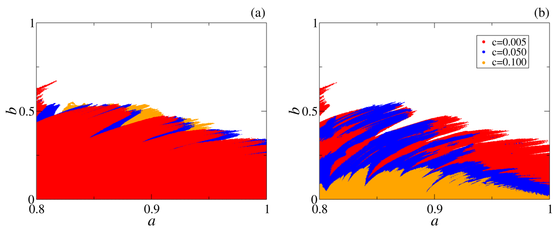
$a = 0.695$ $a = 0.805$

Dividing the escape basins in boxes, we analyze the colors inside each one.

There is a barrier $\rightarrow$ one color, two colors (orange/purple and white) or the three colors (orange, purple and white).

There is not a barrier $\rightarrow$ The same as above with the addition of a new case: a box with just purple and orange.

Parameter spaces



$m = 2$ $m = 4$

	$c = 0.005$	$c = 0.050$	$c = 0.100$
$m = 2$	0.43	0.45	0.42
$m = 4$	0.43	0.28	0.11

Table: Area of the colored region of the parameter space region.

Conclusions

- The escape basins and the analysis of their boundaries is a tool to determine the presence of barriers in the phase space.
- For the extended nontwist map, the addition of a new perturbation can anticipate or postpone the breakup of the barrier.

References

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