Investigation of the chaotic and periodic orbits in the Fermi-Ulam model

Matheus F. M. Magiri, Edson D. Leonel, Juliano A. de Oliveira

Universidade Estadual Paulista (UNESP), Rio Claro, São João da Boa Vista, SP, Brazil

Introduction

The theory of dynamical systems has aroused interest in the studies of many researchers over the last decades [1]. In this work we consider the simplified Fermi-Ulam model which consists of the one-dimensional dynamic of a classical particle of the mass *m* confined between two rigid walls suffering elastic collisions. We define the model by a two-dimensional mapping described by variables velocity *v* and phase of the wall Φ. We investigate the evolution of six initial conditions and observe the almost periodic and chaotic behaviors. We intend to continue development our research studying a description of a complete version of the Fermi-Ulam model and build the phase space to analyze the behavior of the system in the complete version.

Goals

Define the Fermi-Ulam model by a two-dimensional mapping of velocity and phase variables. Investigate the evolution of initial conditions using numerical simulations. Identify almost periodic and chaotic behaviors.

The model

The model we consider is the Fermi-Ulam model described by a two-dimensional mapping in velocity of the particle *v* and phase of the wall Φ variables. The simplified version of the model consists of a classical particle of mass *m* suffering elastic collisions between two rigid, fixed, parallel and horizontal walls. The mapping is defined as [1,2]:

$$\begin{cases} v_{n+1} = |v_n + 2\epsilon \sin(\Phi_{n+1})| \\ \Phi_{n+1} = \Phi_n + \frac{2}{v_n} \mod 2\pi \end{cases}, \qquad (1)$$

where ε is a control parameter that controls the intensity of the nonlinearity. The modulus in the equation of the velocity prevents that a particle not left the region between two walls.

Financial Support

PROPE PRÓ REITORIA DE PESQUISA

CNPq 50 anos

Results

Figure 1 shows the evolution of six different initial conditions using $\varepsilon=10^{-3}$.

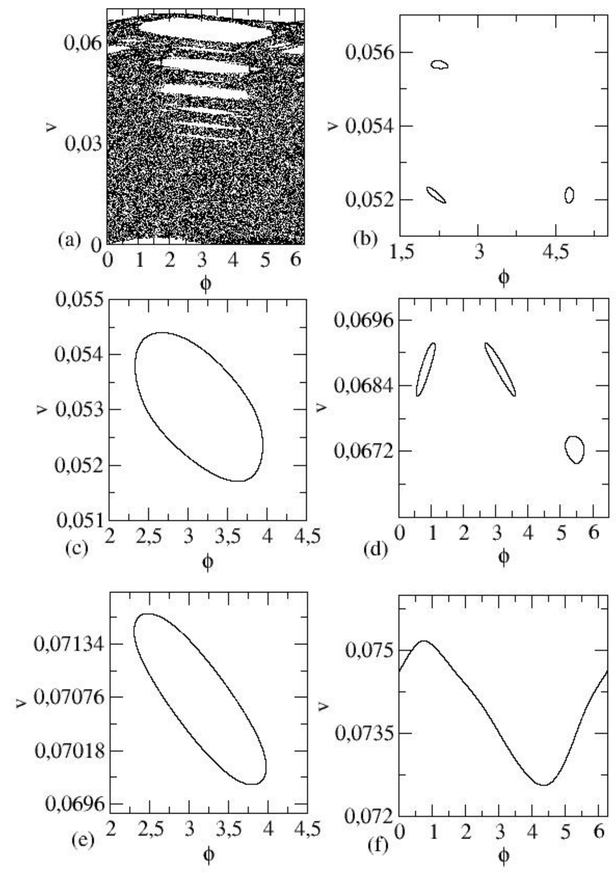

Figure 1: Evolution of the orbits using $\varepsilon=10^{-3}$.

Discussions

Using numerical simutalions we show the evolution of six orbits of the mapping (1). We note that the typical behavior of the chaotic orbits is shown in the figure 1(a) and the behavior of the periodic orbits is shown in figure 1(b)-(f) using different initial conditions. This work is in development and we intend to investigate analyticaly the complete version of the model and to build the phase space to both cases complete and simplified versions to compare the results.

Final Comments

In this work we considered the Fermi-Ulam model and investigated the evolution of six orbits with different initial conditions. Our results have showed typical behaviors of the chaotic and periodic orbits.

References

1 LICHTENBERG, A. J.; LIEBERMAN, M. A.. Regular na chaotic dynamics. 2.ed. V.38. New York: Springer-Verlag, 1992.
2 LEONEL, E. D.; McCLINTOCK, P.V.E. J. Phys. A. 38, 823, 2005.