Dynamical exponents in the dissipative standard mapping

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1. Introduction

The observation of dynamic systems has always fascinated the human being. It is not difficult to come across studies on this subject in Classical Philosophy, Classical Mechanics, among other currents of study, which aimed to understand the evolution of a dynamic system. However, not all systems behaved linearly, that is, predictably. Thus arises the study of nonlinear dynamic systems. The study of nonlinear dynamical systems occurs by observing the behavior of a system in a certain time interval, through the elaboration and development of differential equations. It is noteworthy that these systems do not follow the known algebraic parameters, so in these cases we use the tool called phase space to represent the dynamic behavior of the system through numerical integration. Only after this step is it possible to classify the system as Conservative (when external forces the system is not considered) or Dissipative (considering forces external to the system). The relevance of the research is in its scope, as it can be used to describe problematic situations in various areas of knowledge, such as: electrical, mechanical, biological. It is precisely for these reasons that many scientists have devoted themselves to this subject in recent decades. [1–4]: as the French mathematician Henri Poincaré, [5, 6] and the American meteorologist Edward Lorenz. Finally, the procedure presented in the study can be applied to many systems because its approach is wide. In this paper, we consider the standard Chirikov mapping described by a two-dimensional mapping on moment and angle variables. We built the phase space for the conservative system and observed a mixed structure composed of chaotic sea, periodic islands and a set of invariant curves. The dissipation is introduced into the system and large Chaotic attractors are observed. The maximum chaotic attractors as a function of control parameters is investigated and provide an appropriate energy law. To characterize the chaotic behavior of Lyapunov exponents.

2. Methods

In this work we consider the dissipative standard mapping given by [7]

$$T : \begin{cases} I_{n+1} = (1-\gamma)I_n + k\sin(\theta_n) \\ \theta_{n+1} = \theta_n + I_{n+1} \qquad mod(2\pi) \end{cases}, \tag{1}$$

where I is the momentum variable and θ is the angle variable, γ controls dissipation and k controls the intensity of the nonlinearity. For $\gamma = 0$ recovers the conservative system. Figure 1(a) shows the phase space for the conservative system using the control parameters $\gamma = 0$ and $k = 1$, such that we can observe an chaotic sea and periodic islands. Figure 1(b) shows the phase space for the dissipative system using the control parameters $\gamma = 10^{-3}$ and $k = 100$. For this case, due to dissipation we note that the structure observed to conservative system converges for a large chaotic attractor due the choice of the control parameters.

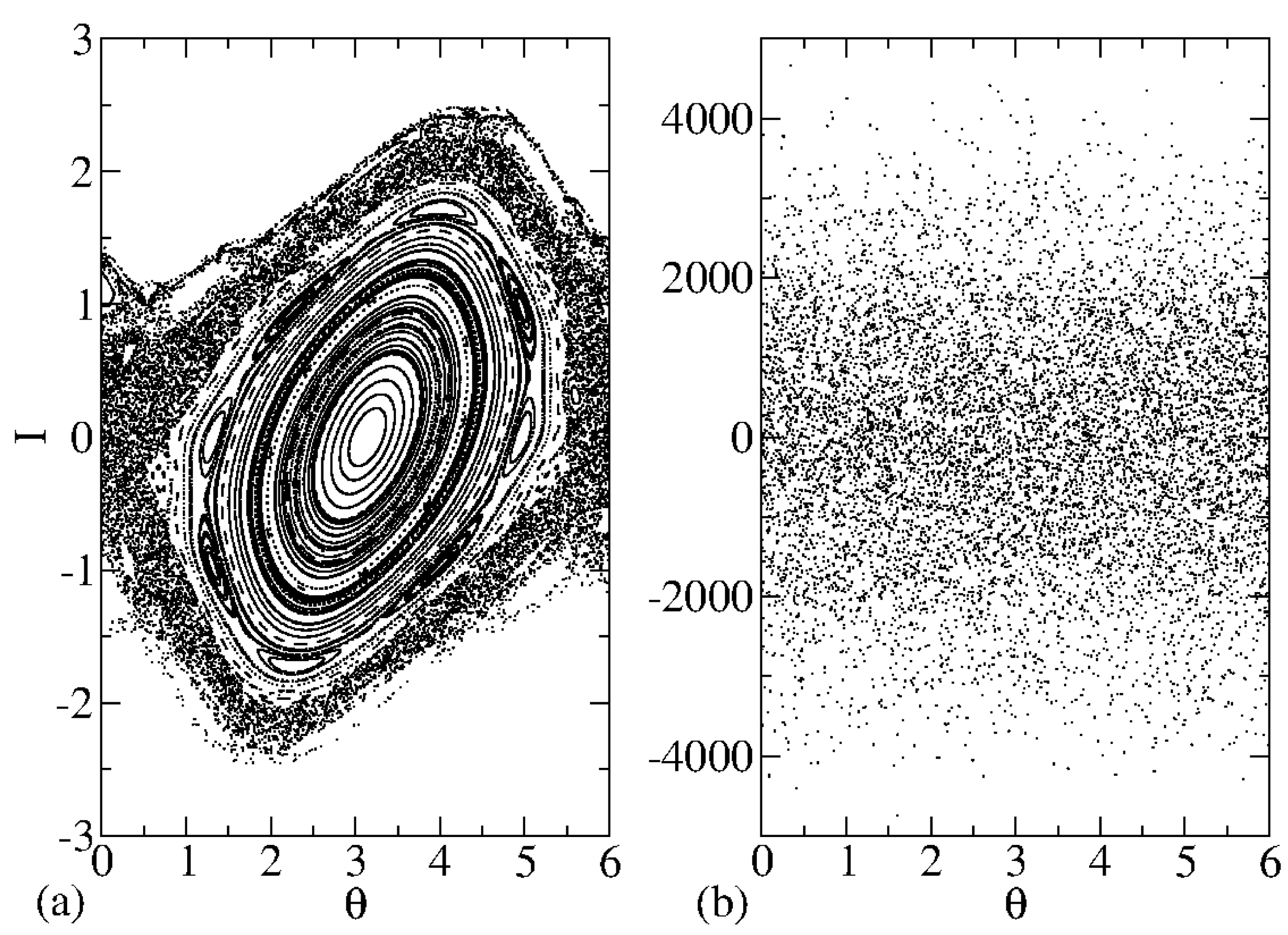

Figure 1: *Phase Space: (a) Conservative system for $k = 1$ and $\gamma = 0$ and (b) Dissipative system for $k = 100$ and $\gamma = 10^{-3}$.*

3. Results

Let us now discuss about the behaviour of the maximum of the chaotic attractors denoted by I^*. Figure 2 shows the behaviour of the I^* as a function of the control parameters k and γ. To build the figure 2(a) we fixed the control parameter $k = 100$ and consider γ in the range $\gamma \in [10^{-5}, 10^{-3}]$. Figure 2(b) we fixed the value of the control parameter $\gamma = 10^{-3}$ and consider k in the range $k \in [10, 10^4]$. In both cases we evaluated the initial conditions 10^8 iterations and obtain the maximum position of the chaotic attractors. A power law fitting provides $\alpha_1 = 1.001(6)$ and $\alpha_2 = -0, 456(7)$ respectively.

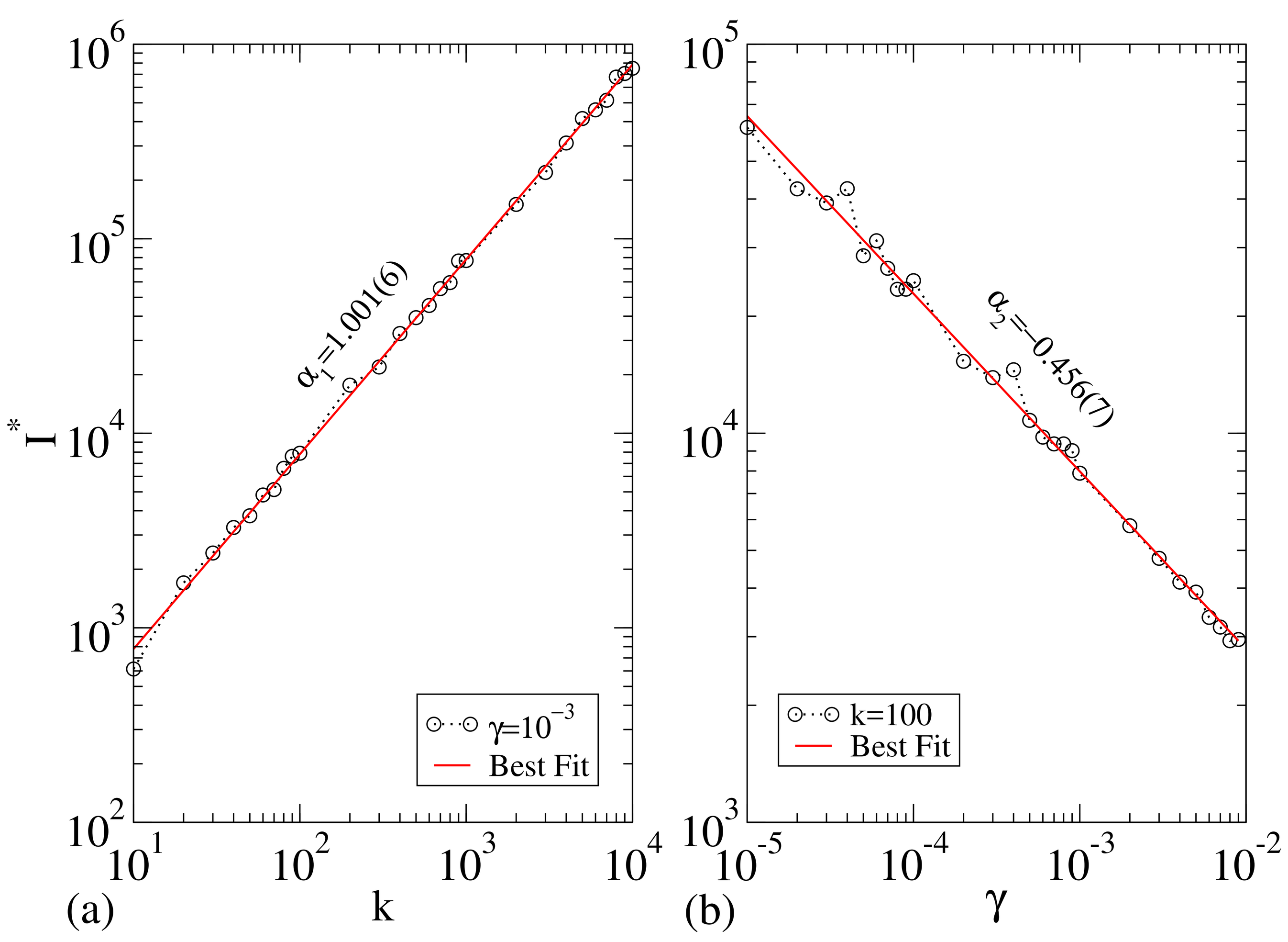

Figure 2: *Maximum value of the chaotic attractor I^*: (a) $\gamma = 10^{-3}$ and $k \in [10, 10^4]$ and (b) $k = 100$ and $\gamma \in [10^{-5}, 10^{-3}]$.*

Now we discuss about the description of the chaotic behaviours. For this, we use the Lyapunov exponents. Basically, this technique measure the divergence rate of the trajectories. When there is at least one $\lambda > 0$ (positive value) the orbital system is chaotic, for $\lambda < 0$ (negative value) the system is described with periodic orbit. The Lyapunov exponents are defined as

$$\lambda_j = \lim_{n\to\infty} \frac{1}{n} \ln |\Lambda_j^{(n)}|, \quad j = 1, 2, \tag{2}$$

where $\Lambda_j^{(n)}$ are the eigenvalues of the matrix $M = \Pi_{i=1}^{n} J_i(\theta, I)$ where J_i is the Jacobian matrix of the mapping evaluated along the orbit.

Figure 3 shows the behavior of the Lyapunov exponents using two different values of k and γ for five different initial conditions as labeled in the figure. Figure 3(a) shows the behavior for $k = 100$ and $\gamma = 10^{-3}$, while figure 3(a) shows the behavior for $k = 1000$ and $\gamma = 10^{-4}$. The average of the convergence of the Lyapunov exponents provides $\bar{\lambda}_1 = 3.912(1)$, where the error 0.001 corresponds to a standard deviation of the five sample and $\bar{\lambda}_1 = 6.2144(8)$, where the error 0.0008 corresponds to the standard deviation respectively.

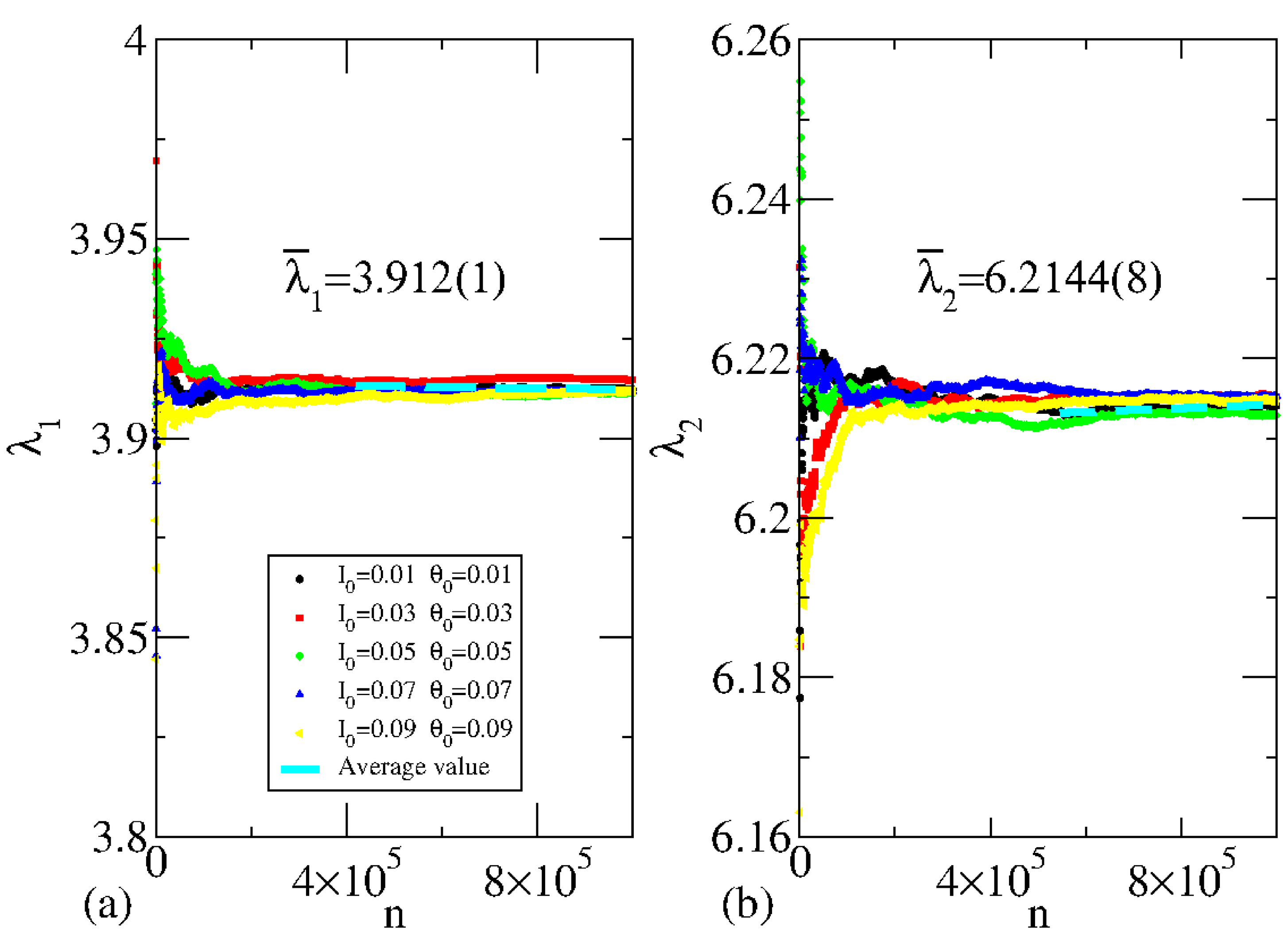

Figure 3: *Lyapunov Exponent: (a) $k = 100$ and $\gamma = 10^{-3}$ and (b) $k = 10000$ and $\gamma = 10^{-4}$.*

4. Final comments

Some properties of the standard map were presented in this work, among them the conservative case and the dissipative case; because these properties describe our nonlinear system, the pulsed rotor. With the aid of computer programs it was possible to show the evolution of orbits and build the phase space, in order to classify them as conservative or dissipative. Such first fruits allowed us to present the maximum chaotic attractors and consequently the critical exponent of our model. In this line, the work investigated and determined the average of Lyapunov exponents and thus, as our exponents are positive, the intensity of chaotic orbits in the mapping was evidenced.

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