EXPLOSIVE SYNCHRONIZATION GENERATED BY THE ADDITION OF NON-LOCAL CONNECTIONS IN NEURAL NETWORKS

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INTRODUCTION

Explosive synchronization (ES) has recently been observed in complex networks with chaotic oscillators and a frequency–connectivity-degree correlation (GÓMEZ-GARDENES et al., 2011; HESSE; GROSS, 2014). Here, we investigate the mechanism for ES transition of a complex neural network composed of non-identical two-dimensional maps coupled by a Newman-Watts small-world vicinity matrix (NEWMAN; WATTS, 1999). We find a range of probabilities in which the network displays an abrupt transition to phase synchronization, characterizing an ES. The mechanism behind the onset of ES is the following: as the coupling parameter is increased adiabatically, ES is likely to occur as a transition from a chaotic non-synchronized to a regular phase-synchronized asymptotic state, this transition occurring in $\varepsilon = \varepsilon^\dagger$ is namely frontier crisis where the chaotic attractor collides with its attraction basin boundary (OTT, 2002; GREBOGI; OTT; YORKE, 1986). As the coupling parameter is adiabatically decreased at $\varepsilon = \varepsilon^*$ the periodic attractor loses stability and characterizes a saddle-node bifurcation (OTT, 2002).

NEURON MODEL

We consider a network composed of $N = 10\,000$ nodes where the local dynamics is given by the neural model proposed by Chialvo (CHIALVO, 1995)

$$x_{i,t+1} = x_{i,t}^2 \exp(y_{i,t} - x_{i,t}) + k_i + \frac{\varepsilon}{\eta}\sum_{j=1}^{N} e_{i,j}x_{j,t}, \quad (1)$$

$$y_{i,t+1} = a y_{i,t} - b x_{i,t} + c, \quad (2)$$

where $x_{i,t}$ and $y_{i,t}$ are the activation and recovery variables. k_i acts as a constant bias or as a time-dependent additive perturbation (CHIALVO, 1995), which here is supposed to vary randomly between $[k_0, k_0 + \sigma]$ being σ the coefficient of dissimilitude. a, b, and c are parameters of the model, η is a normalization factor given by the average number of connections in the network, ε is the coupling parameter and $e_{i,j}$ are elements of the network connection matrix.

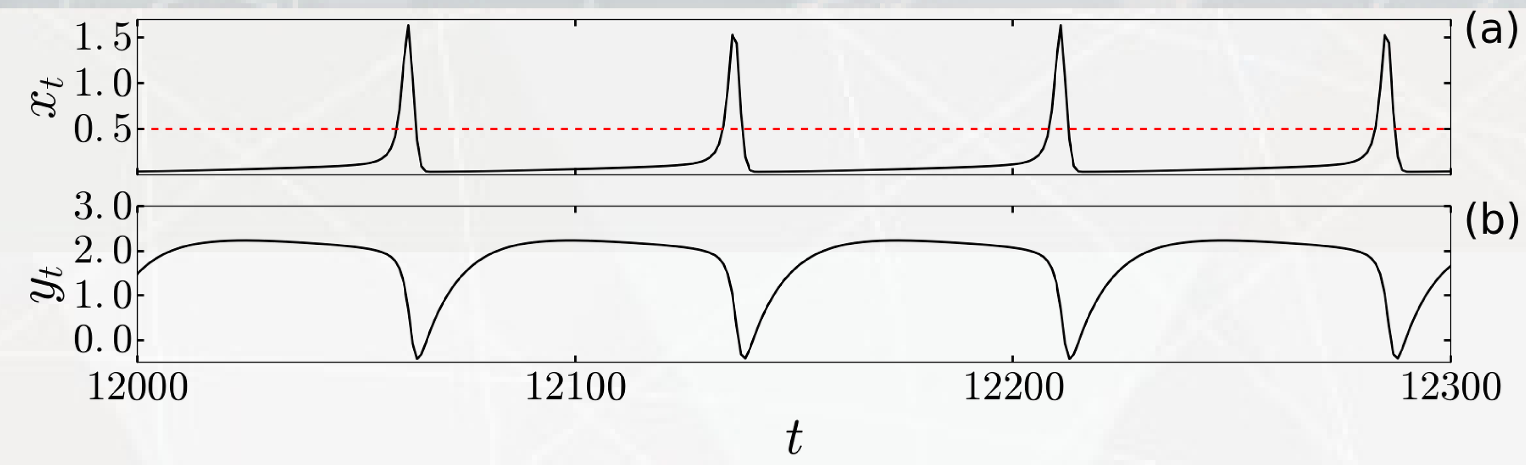

Figure 1 – Dynamical behavior of the neuron model for $a = 0.89$, $b = 0.6$, $c = 0.28$, and $k = 0.03$, (a) x_t and (b) y_t dynamics. The red dashed line in panel (a) defines a Poincaré surface in $x_0 = 0.5$ used to evaluate the beginning of each spike and the associated phase of the neuron.

SYNCHRONIZATION QUANTIFIER

In order to quantify phase synchronization we define a Poincaré's surface at $x_0 = 0.5$ to evaluate the time of the beginning of each spike. In panel (a) of the Fig. 1 depicts a red dashed line which correspond to this Poincaré's surface, so each time that a $x_{i,t}$ reaches x_0 (upwards) $\theta_i(t)$ is increased by 2π, such that the interpolation of the time varying phase is defined by (IVANCHENKO et al., 2004; BOCCALETTI et al., 2002)

$$\theta_i(t) = 2\pi\ell_i + 2\pi\frac{t - t_{\ell,i}}{t_{\ell+1,i} - t_{\ell,i}}, \quad t_{\ell,i} \le t < t_{\ell+1,i}, \quad (3)$$

where ℓ_i is the ℓ^{th} spike of the i^{th} neuron, t is the current time, and $t_{\ell,i}$ is the time for which the i^{th} neuron starts the ℓ^{th} spike.
The synchronization of the maps is evaluated by using the modulus of the Kuramoto's order parameter $R(t)$ (KURAMOTO, 2012)

$$R(t) = \left|\frac{1}{N}\sum_{i=1}^{N} e^{i\theta_i(t)}\right|, \quad (4)$$

where θ_i is the phase of the i^{th} neuron at the time t, defined by Eq.(3). $R(t)$ quantifies in a number the behavior of the network syncrhonization since $R \to 0$ for complete unsynchronized state and $R \to 1$ for complete phase synchronized state.

RESULTS

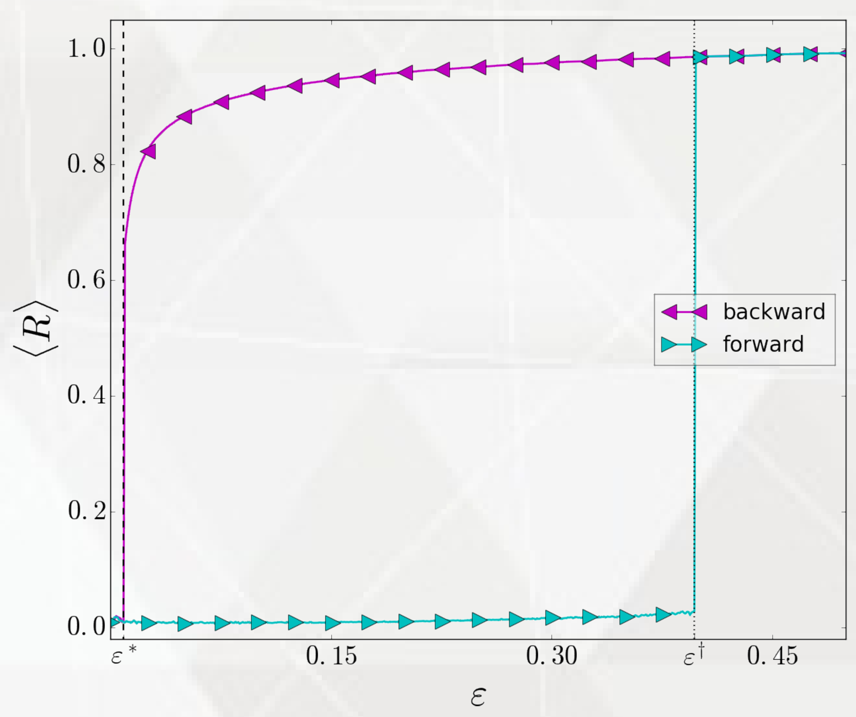

Figure 2 – $\langle R \rangle$ as a function of the coupling between the maps for a fixed probability $p_{\text{nl}} = 0.15$ and $\sigma = 0.001$. The coupling is evolved adiabatically ($\delta\varepsilon = 0.001$) in two different directions: forward and backward. In forward direction when ε reaches a critical value $\varepsilon^\dagger$ the network synchronize spontaneously. In the backward direction the network loses the synchronized state when $\varepsilon = \varepsilon^*$ closing the hysteretic loop $\varepsilon^* < \varepsilon < \varepsilon^\dagger$.

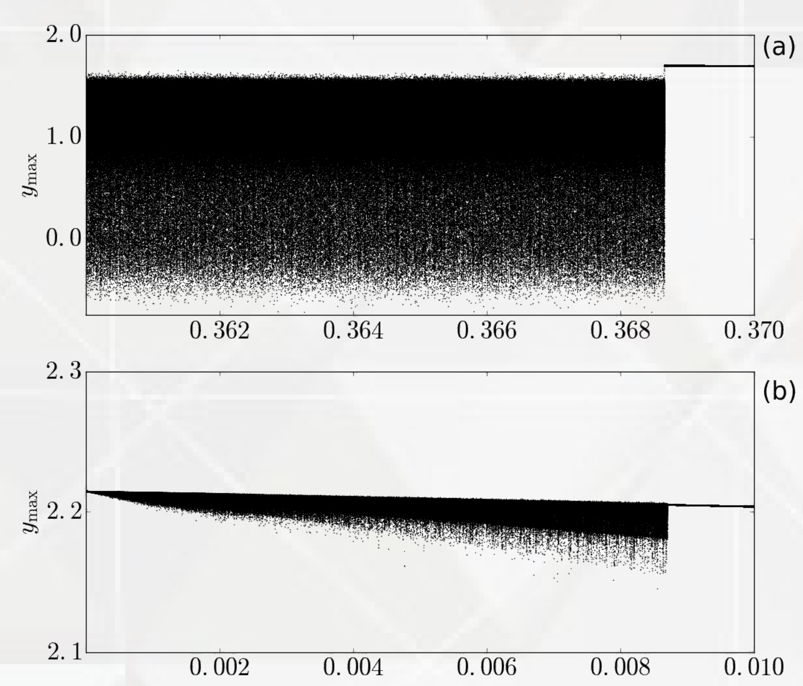

Figure 3 – Bifurcation diagram of the $y_{\max}$ for a i^{th} of network as a function of the coupling. (a) Evolution in the forward direction. (b) Evolution in the backward direction. In panel (a) the neuron exhibits a chaotic behavior until $\varepsilon^\dagger$ where the chaotic attractor collides with its attraction basin boundary (OTT, 2002). In panel (b) the neuron exhibits periodic behavior until ε^* in which stable and unstable orbits coalesce and obliterate each other, this transition is called saddle-node bifurcation (OTT, 2002).

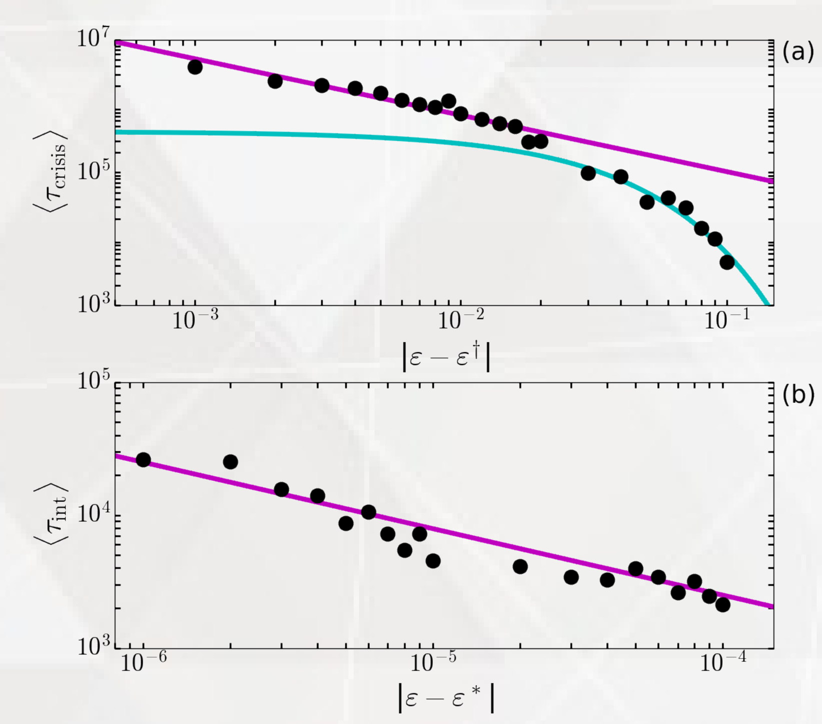

Figure 4 – In a frontier crisis, the post-crisis behavior ($\varepsilon \gtrsim \varepsilon^\dagger$) the chaotic attractor is replaced by a chaotic saddle which allows a chaotic transient τ_{crisis} for initial conditions initialized inside the attractor (OTT, 2002). (a) Characteristic time of chaotic transient $\langle\tau_{\text{crisis}}\rangle$ as a function of the distance of the critical coupling $\varepsilon^\dagger$. The solid curve corresponds to a theoretic curve at $\langle\tau_{\text{crisis}}\rangle \propto |\varepsilon - \varepsilon^\dagger|^{-0.85}$. In a saddle-node bifurcation, the stable non-chaotic state loses stability. In such a transition, intermittency is always observed due to the *quasi*-stable character of the synchronized state after the bifurcation. (b) Characteristic time of intermittency $\langle\tau_{\text{int}}\rangle$ as a function of $|\varepsilon - \varepsilon^*|$. The solid curve corresponds to a theoretic curve at $\langle\tau_{\text{int}}\rangle \propto |\varepsilon - \varepsilon^*|^{-1/2}$.

CONCLUSIONS

- It is shown that the network presents explosive synchronization followed by an hysteretic loop, where exist a bi-stable regime for $\varepsilon^* < \varepsilon < \varepsilon^\dagger$.
- The transition from non-synchronized to synchronized state (at $\varepsilon = \varepsilon^\dagger$) is characterized by a chaotic transition where the attractor gains stability through a frontier crisis.
- In the transition from synchronized to non-synchronized state (at $\varepsilon = \varepsilon^*$) the network loses stability by a saddle-node bifurcation.

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