Effects of the leak conductance in the synchronization of a Hodgkin-Huxley type network.

Perspectives on Oscillation Control

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Abstract
In this work, we studied how the distinction of the leak conductance in the Huber and Braun *et al.* model affects the synchronization of a neuronal network. We simulated the distinction of the parameter using a truncated gaussian distribution, following the dynamical limits of the conductance.

Introduction

In 1952 Alan Lloyd Hodgkin and Andrew Huxley proposed a mathematical model for the action potential of a neuron [1], which later awarded them with the Nobel Prize in Phisiology. The main equation is:

$$C_M\frac{\mathrm{d}V}{\mathrm{d}t} = -I_K - I_{Na} - I_L + I_{ext}.$$

The equation comes from the capacitor equation, with C_M being the membrane capacitance, V is the associated potential, and I_K, I_{Na}, I_L and I_{ext} are, the current associated with the potassium ion, the sodium ion, the leak current and an external current, respectively. The currents can be written as

$$\begin{aligned}I_K &= \bar{g}_K n^4(V - E_K),\\ I_{Na} &= \bar{g}_{Na} m^3 h(V - E_{Na}),\\ I_L &= \bar{g}_L(V - E_L),\end{aligned}$$

where $\bar{g}_K$, $\bar{g}_{Na}$ and $\bar{g}_L$ are the associated conductancies, and E_K, E_{Na}, E_L represents the rest potential from each ion. The parameters m, n and h are experimentally obtained.

The Huber-Braun equations

The Hodgkin-Huxley model is not capable to reproduce the bursting of spikes dynamics of a neuron, so Huber and Braun *et al.* [2] modelled the equations to be thermally sensitive, with addition of two currents , I_{sd} and I_{sa}, that are responsible for the activation when the potential is bellow a threshold. The new set of equations are:

$$C_M\frac{\mathrm{d}V}{\mathrm{d}t} = -I_K - I_{Na} - I_{sd} - I_{sa} - I_L + I_{ext},$$

and the new currents with temperature dependence:

$$\begin{aligned}I_K &= \rho\bar{g}_K a_K(V - E_K),\\ I_{Na} &= \rho\bar{g}_{Na} a_{Na}(V - E_{Na}),\\ I_{sd} &= \rho\bar{g}_{sd} a_{sd}(V - E_{sd}),\\ I_{sa} &= \rho\bar{g}_{sa} a_{sa}(V - E_{sa}),\\ I_L &= \bar{g}_L(V - E_L),\end{aligned}$$

where a_K, a_{Na}, a_{sd}, a_{sa}, are ionic channels opening probabilities and ρ is a function with temperature dependence.

In a network with N neurons, coupled with a Small-World adjacency matrix, the Huber-Braun equations to the *i-th* neuron, is

$$C_M\frac{\mathrm{d}V^{(i)}}{\mathrm{d}t} = -I_K^{(i)} - I_{Na}^{(i)} - I_{sd}^{(i)} - I_{sa}^{(i)} - I_L^{(i)} + I_{coup}^{(i)},$$

with

$$I_{coup}^{(i)} = \frac{\epsilon}{n}\sum_{j=1}^{N} e_{(ij)} r^{(j)}(V_{sin} - V^{(i)}),$$

the $e_{(ij)}$ are the adjacency matrix elements, ϵ is the coupling parameter, V_{sin} is a sinaptic potential, n is the normalization from the most connected neuron and $r^{(j)}$ is a differential equation that simulates the channel opening. In Figure 1 we can see how the coupling parameter may affect the bursting of neurons.

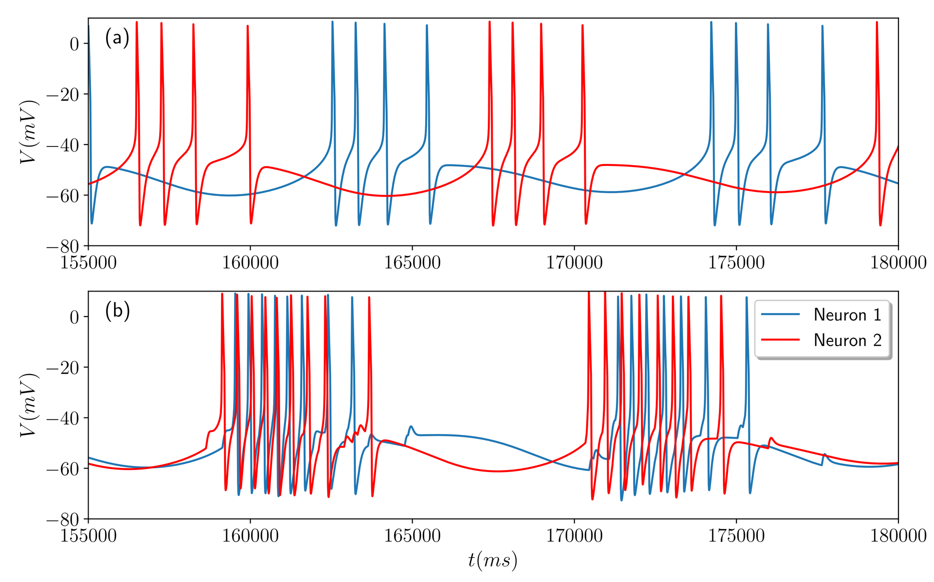

Figure 1: Bursting neurons regime with (a) $\epsilon = 0.0$ and (b) $\epsilon = 0.08$.

Phase synchronization

A phase $\theta^{(j)}(t)$ can be associated with the *j-th* bursting neuron with t as the time of integration, $t_k^{(j)}$ as the time of the *k-th* neuronal spike and $t_{k+1}^{(j)}$ as the following spike.

$$\theta^{(j)}(t) = 2\pi k^{(j)} + 2\pi\frac{t - t_k^{(j)}}{t_{k+1}^{(j)} - t_k^{(j)}},$$

The Kuramoto order parameter [3] can be associated with this phase

$$z(t) = R(t)e^{i\psi(t)} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta^{(j)}(t)},$$

where $R(t)$ is the modulus of $z(t)$ and N is the number of oscillators. The temporal average of the parameter can be obtained with

$$\langle R\rangle = \frac{1}{t_f - t_i}\sum_{t=t_i}^{t_f} R(t).$$

Results

In this work, we numerically simulated the synchronization of a small-world network with $N = 512$ neurons. First, we calculated the bifurcation diagram of the interspike interval (*ISI*) so we can understand what is the dynamics of the leak conductance (Figure 2). We observed that for values greater than $\bar{g}_L = 0.15$ there is no bursting or spike regime, so for our parameter space analysis, we limited the distributions to values of $0.06 \leq \bar{g}_L \leq 0.15$. In Figure 3 we simulated truncated gaussian distributions, where σ is the standard deviation, to several values of the leak conductance and observed the synchronization pattern of the network when we forced the coupling parameter.

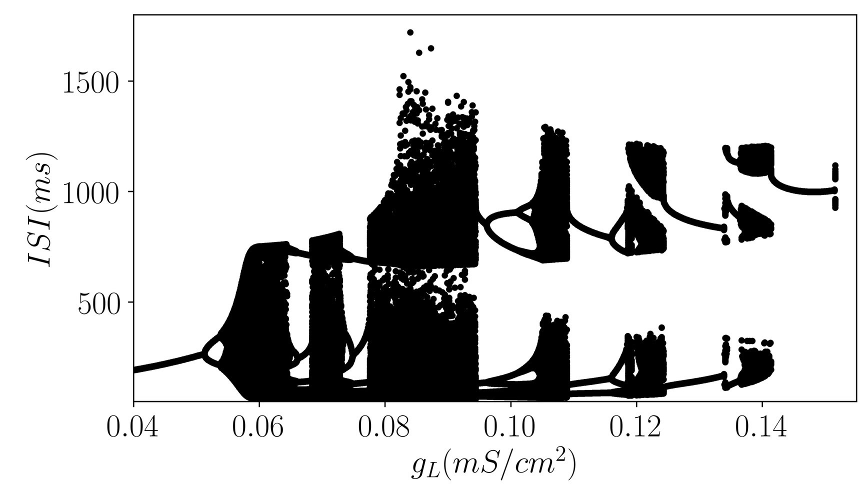

Figure 2: Bifurcation diagram of the interspike interval for a Poincaré surface at $-20\,mV$.

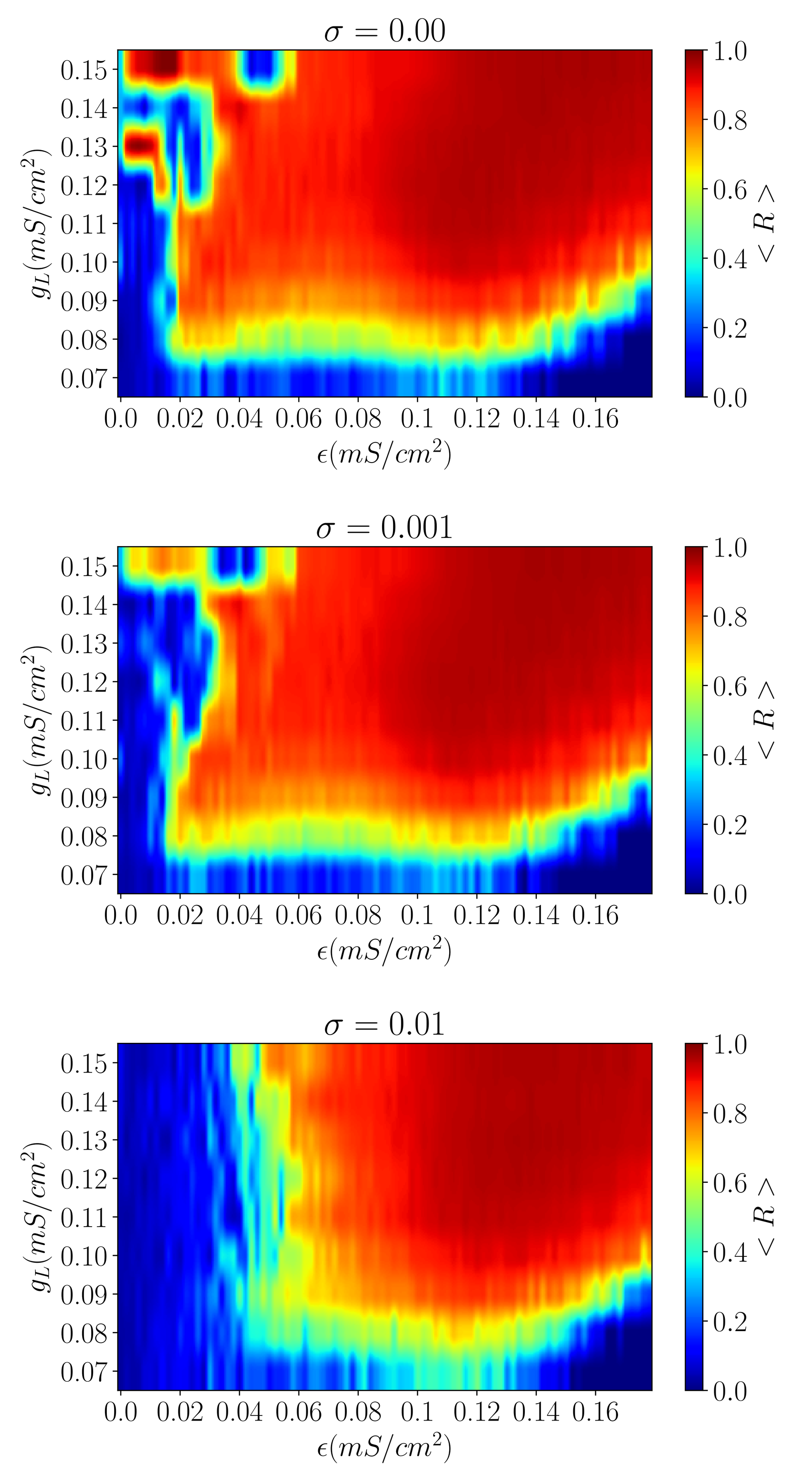

Figure 3: Parameter space for different values of standard deviation of the gaussian distribution for the leak conductance. With $\sigma = 0.00$ a network when the conductance is fixed.

Conclusions

A small distinction of the leak conductance, such as $\sigma = 0.001$, may not affect values bigger than $\epsilon = 0.03$ of the coupling strength, but it can affect lower values. However, for $\sigma = 0.01$, it affects the network as a whole, slowering the synchronization process and eliminating all of the synchronized regions with lower values of ϵ.

References

[1] A. L. Hodgkin and A. F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *The Journal of Physiology*, 117(4):500–544, 1952.

[2] U. Feudel, A. Neiman, X. Pei, W. Wojtenek, H. Braun, M. Huber, and F. Moss. Homoclinic bifurcation in a hodgkin-huxley model of thermally sensitive neurons. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 10(1):231–239, 2000.

[3] Y. Kuramoto. *Chemical oscillations, waves, and turbulence*, volume 19. Springer Science & Business Media, 2012.

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