Chaos and stability analysis in the perturbed logistic-like map

Anderson A. A. da SILVA*[1], Edson D. LEONEL[2], Juliano A. de OLIVEIRA[3]

[1]UNESP - *campus* Rio Claro

[2]UNESP- *campus* Rio Claro

[3]UNESP - *campus* São João da Boa Vista

⋆andersonantsilva1@gmail.com

Introduction

The study of dynamical systems has been attracting the interest of many researchers in recent years [1, 2, 3, 4]. Among the several models investigated in the literature we highlight the logistic map which is a deterministic equation used to describe population dynamics[5]. Over the years the interest in studying such a map only increases and one of the reasons is that this equation, although simple, presents an interesting and complex behavior. In the present work we investigate some dynamical properties in the logistic-like map which is parameterized by an exponent and consider a parametric perturbation[6, 7].

Results and Discussions

The perturbed logistic-like map is described as

$$x_{n+1} = R(1+\epsilon\cos(n\pi))x_n(1-x_n^{\gamma}), \quad (1)$$

where R and γ are control parameters so that $\gamma > 0$. For $\epsilon = 0$ we return to the logistic-like map, while for $\epsilon \neq 0$ we define the perturbed logistic-like map. The bifurcation diagram of the map (1) shows that as the parameter R varies the attractors appear and change the stability. Since R oscillates between two values $R(1+\epsilon)$ and $R(1-\epsilon)$, the logistic-like map will have two periodic fixed points in time (n). The fixed points of the equation (1) are obtained from condition $x_{n+1} = x_n = x^*$.

Depending of the initial condition the orbits in bifurcation diagram changes of the attraction basins in some points Rc. The values of R_c are estimated numerically by the Newton Raphson method providing $R_c = 3.14439$ for $\gamma = 1$ and $R_c = 2.15294$ for $\gamma = 2$. To characterize the chaos we use the Lyapunov exponents. The idea to obtain the Lyapunov exponents consists of checking for exponential divergence of nearby trajectories when time grows up. If the trajectories stay nearby during all the time or convergence, the system is not sensitive to initial conditions, so $\lambda \leq 0$ implies periodic asymptotic motion. If the nearby trajectories between initial conditions grow up exponentially with time, we have sensibility to the initial conditions. Thus, only one positive exponent $(\lambda > 0)$ is sufficient to produce chaos in the systems. The Lyapunov exponents are defined as follow: Given an one-dimensional map

$$x_{n+1} = F(x_n), \quad (2)$$

we consider two initial points x_0 and y_0 and the distance between them given by:

$$\delta = |y_0 - x_0|. \quad (3)$$

After an iteration the new distance is:

$$\delta' = |y_1 - x_1|, \quad (4)$$

where $\delta = e^L\delta$, and L measures the rate of divergence of expansion δ even the distance δ'. After some algebric manipulations and iterating the map N times we can rewrite the equation as:

$$L = \frac{1}{N}ln\left|\frac{F^N(x_0+\delta) - F^N(x_0)}{\delta}\right|. \quad (5)$$

Rewriting the equation above, we have

$$\lambda(x_0) \equiv L(x_0) = \lim_{N\to\infty}\lim_{\delta\to 0}\frac{1}{N}ln\left|\frac{F^N(x_0+\delta) - F^N(x_0)}{\delta}\right|, \quad (6)$$

and obtain the Lyapunov exponent defined as:

$$\lambda(x_0) = \lim_{N\to\infty}\frac{1}{N}\sum_{i=0}^{N-1} ln|F'(x_i)|, \quad (7)$$

where $|F'(x_i)|$ represents the first derivative of de map (1). In the figure1, we will illustrate an idea of lyapunov exponent.

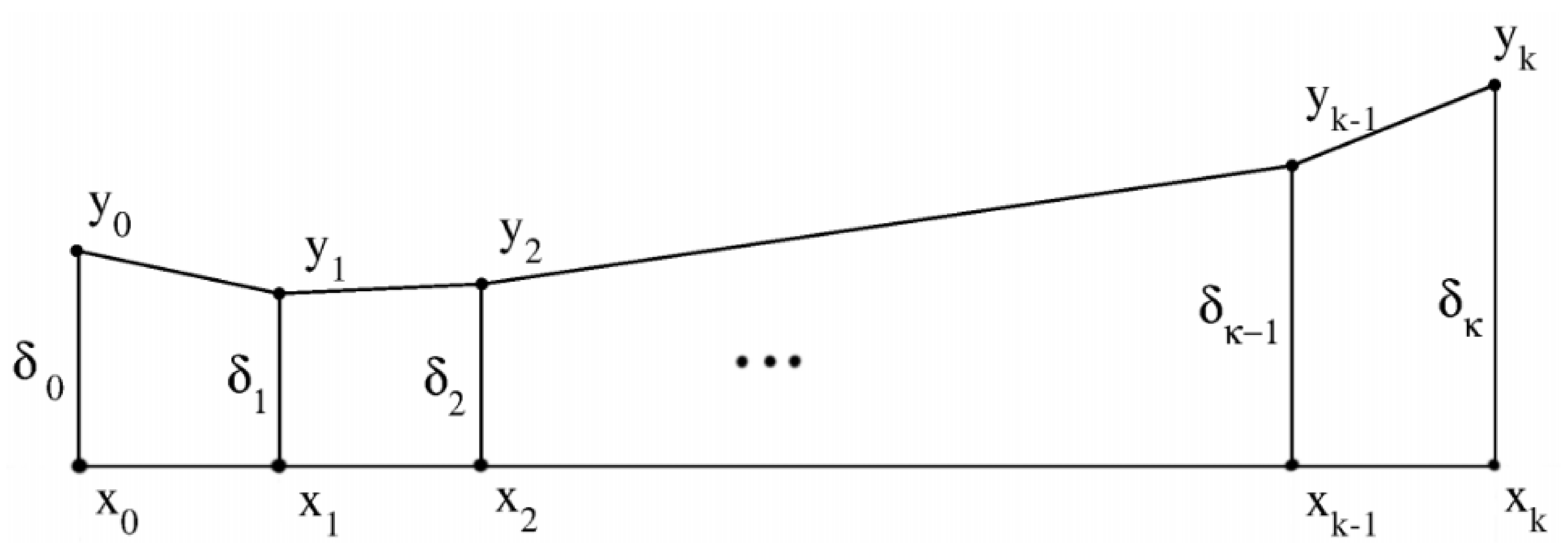

Figure 1: *x_0 and y_0 are neighboring initial conditions and δ_0 is an infinitesimal distance.*

Figure 2 shows the fork diagram along with the Lyapunov exponent considering $\gamma = 1$ and $\epsilon = 0.01$ and different values for initial condition x_0 according to figure legend.

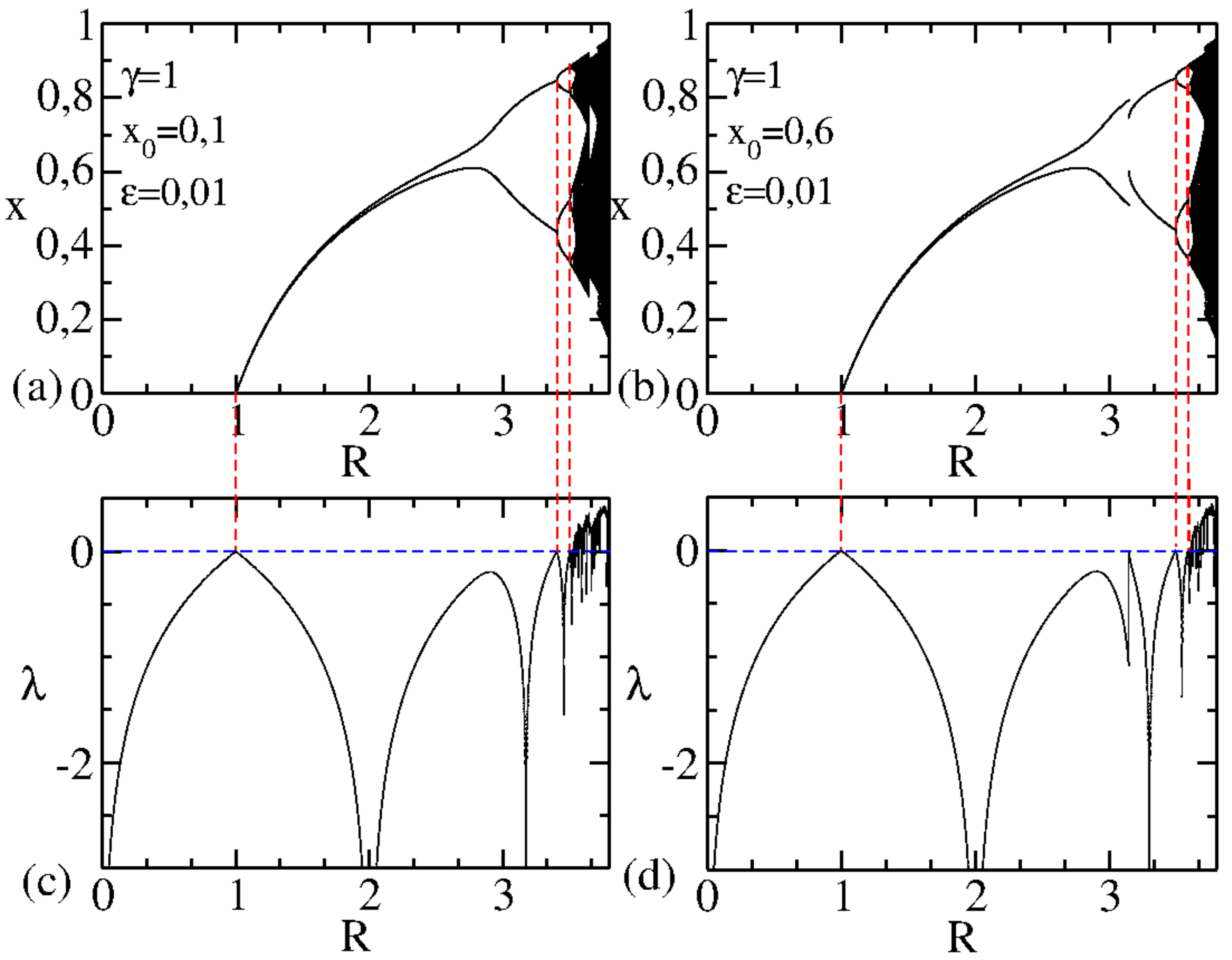

Figure 2: *Bifurcations diagram along with their respective Lyapunov exponents.*

Conclusions

In this work, we consider the logistic-like map perturbed, we observe that for different values of initial condition x_0, a discontinuity appears when we approach a critical value called R_c, we find these values using the Newton Raphson method. We also present the Lyapunov exponent which we calculate numerically.

References

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