Symplectic Euler method

Be a Hamiltonian function $H = H(\theta, p)$ with the respective equations of motion,

$$\dot{\theta} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial \theta}. \tag{1}$$

The numerical integration by the Symplectic Euler method is made by the iteration of the equations [1],

$$\begin{aligned} \theta_{n+1} &= \theta_n + h\, H_p(p_n, \theta_{n+1}), \\ p_{n+1} &= p_n - h\, H_\theta(p_n, \theta_{n+1}), \end{aligned} \tag{2}$$

where h is the step and the function H_p and H_θ are defined by,

$$H_p = \frac{\partial H}{\partial p}, \qquad H_\theta = \frac{\partial H}{\partial \theta}. \tag{3}$$

The Hamiltonian function that describe the system is defined by,

$$\mathcal{H}(I, \theta, \phi) = I\left(1 - \frac{I}{4}\right) - 2\,\varepsilon\, I^{m/2} \cos(m\theta) f(\phi), \tag{4}$$

with,

$$f(\phi) = \begin{cases} 1, \text{ if } 0 \leq \phi < \xi, \\ 0, \text{ if } \xi \leq \phi < 2\pi. \end{cases} \tag{5}$$

With equations (2)-(5), we obtain the iteration equations

$$\begin{aligned} \theta_{n+1} =& \theta_n + h\left[1 - \frac{I_n}{2} - m\,\varepsilon\, I_n^{m/2-1}\ \cos(m\,\theta_{n+1})\ f(\phi)\right], \\ I_{n+1} =& I_n - h\left[2\,m\,\varepsilon\, I_n^{m/2} \sin(m\,\theta_{n+1})\ f(\phi)\right]. \end{aligned} \tag{6}$$

In order to obtain θ_{n+1}, we apply the Newton-Rhapson method in the first equation in (6).

The implementation of the numerical integration was made in FORTRAN and the code is presented below and available in the repository `http://web.if.usp.br/controle/`.

References

[1] E. Hairer and G. Wanner, "Euler methods, explicit, implicit, symplectic," *Encyclopedia of Applied and Computational Mathematics*, vol. 1, pp. 451–455, 2015.