Ergodic Magnetic Limiter

■ Components of the magnetic field produced by an EML (neglecting border effects)

$$\begin{aligned} B_r^{(1)}(r,\theta,\phi) &= -\frac{\mu_0\, m\, I_L}{\pi\, a^m} r^{m-1}\sin(m\theta) f(\phi), \\ B_\theta^{(1)}(r,\theta,\phi) &= -\frac{\mu_0\, m\, I_L}{\pi\, a^m} r^{m-1}\cos(m\theta) f(\phi), \end{aligned} \tag{1}$$

where,

$$f(\phi) = \begin{cases} 1, & \text{if} \quad 0 \le \phi < \ell/R_0, \\ 0, & \text{if} \quad \ell/R_0 \le \phi < 2\pi, \end{cases} \quad = \quad \frac{\ell}{2\pi\, R_0}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\phi)\right\}. \tag{2}$$

■ **Hamiltonian function for the EML**

The Hamiltonian is calculated by $H = \int \frac{dJ}{q(J)}$, where the safety factor profile is computed by $q(r) = \frac{r\, B_0}{R_0\, B_\theta(r)}$. From Equation (1),

$$q(r) = -\frac{\pi\, a^m\, B_0}{\mu_0\, m\, I_L\, R_0\, r^{m-2}\, \cos(m\theta)\, f(\phi)}. \tag{3}$$

The non-canonical transformation states that $J = r^2/2$. So, $q(J)$ is defined by,

$$q(J) = -\frac{\pi\, a^m\ B_0}{\mu_0\, m\, R_0\, I_L\, (2\,J)^{\frac{m-2}{2}}\cos(m\theta)\, f(\phi)}, \tag{4}$$

and the Hamiltonian function related to the limiter is,

$$H_1 = \int \frac{d\,J}{q(J)} = -\frac{\mu_0\, m\, R_0\, I_L\, \cos(m\theta)\, f(\phi)}{\pi\, a^m\, B_0}\int (2\,J)^{\frac{m-2}{2}}\, dJ \longrightarrow H = -\frac{\mu_0\, R_0\, I_L}{\pi\, a^m\, B_0}(2\,J)^{m/2}\cos(m\theta) f(\phi) \tag{5}$$

From Equation (2), we can rewrite the Hamiltonian as,

$$H_1 = -\frac{\mu_0\, \ell\, I_L}{2\pi^2 B_0}\frac{(2\,J)^{m/2}}{a^m}\left\{\cos(m\theta) + 2\sum_{n=1}^{\infty}\cos(m\theta)\cos(n\phi)\right\} \tag{6}$$

Defining,

$$\sigma = \frac{\mu_0\, I_L\, \ell}{2\pi^2\, B_0} \qquad \text{and} \qquad A_m(J) = \frac{(2\,J)^{m/2}}{a^m}, \tag{7}$$

and using the relation $2\cos a\cos b = \cos(a+b) + \cos(a-b)$, the Hamiltonian is,

$$H_1 = H(J,\theta,\phi) = -\sigma A_m(J)\left\{\cos(m\theta) + \sum_{n=1}^{\infty}\left[\cos(m\theta - n\phi) + \cos(m\theta + n\phi)\right]\right\} \tag{8}$$