Clebsch representation

The vector potential in magnetic flux coordinates is written as,

$$\mathbf{A} = A_\theta \nabla\theta + A_\psi \nabla\psi + A_\zeta \nabla\zeta. \tag{1}$$

Let a scalar function be defined by,

$$G = \int A_\psi \, d\psi, \quad \longrightarrow \quad \frac{\partial G}{\partial \psi} = A_\psi \tag{2}$$

then, its gradient is,

$$\nabla G = \frac{\partial G}{\partial \theta}\nabla\theta + \frac{\partial G}{\partial \psi}\nabla\psi + \frac{\partial G}{\partial \zeta}\nabla\zeta \quad \longrightarrow \quad \boxed{\nabla G = \frac{\partial G}{\partial \theta}\nabla\theta + A_\psi \nabla\psi + \frac{\partial G}{\partial \zeta}\nabla\zeta}\,. \tag{3}$$

Calculating $A - \nabla G$, we have

$$\mathbf{A} - \nabla G = \left(A_\theta - \frac{\partial G}{\partial \theta}\right)\nabla\theta + \left(A_\zeta - \frac{\partial G}{\partial \zeta}\right)\nabla\zeta. \tag{4}$$

Defining,

$$\psi = A_\theta - \frac{\partial G}{\partial \theta}, \qquad \alpha = -A_\zeta + \frac{\partial G}{\partial \zeta} \tag{5}$$

the vector $\mathbf{A}$ is,

$$\mathbf{A} = \nabla G + \psi\nabla\theta - \alpha\nabla\zeta. \tag{6}$$

The magnetic field in terms of the flux is obtained by the rotational $\mathbf{B} = \nabla \times \mathbf{A}$. In these terms,

$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla \times (\nabla G + \psi\nabla\theta - \alpha\nabla\zeta) \quad \rightarrow \quad \mathbf{B} = \nabla \times \nabla G + \nabla \times (\psi\nabla\theta) - \nabla \times (\alpha\nabla\zeta). \tag{7}$$

From vectors identities, for a vector $\mathbf{V}$ and a scalar f, we have,

$$\nabla \times (f\,\mathbf{V}) = f\,\nabla \times \mathbf{V} + (\nabla f) \times \mathbf{V} \tag{8}$$

and $\nabla \times \nabla f = 0$. With these,

$$\mathbf{B} = \cancelto{0}{\nabla \times \nabla G} + [\psi\, \cancelto{0}{\nabla \times (\nabla\theta)} + (\nabla\psi) \times \nabla\theta] - [\alpha \cancelto{0}{\nabla \times (\nabla\zeta)} + (\nabla\alpha) \times \nabla\zeta] \tag{9}$$

$$\boxed{\mathbf{B} = \nabla\psi \times \nabla\theta - \nabla\alpha \times \nabla\zeta} \tag{10}$$